

DEVELOPMENT OF FLEXURE TESTING FIXTURES AND  
METHODS FOR THIN-WALLED COMPOSITE TUBES

by

Bryce Ingersoll

A thesis submitted to the faculty of  
The University of Utah  
in partial fulfillment of the requirements for the degree of

Master of Science

Department of Mechanical Engineering

The University of Utah

December 2010

Copyright © Bryce Ingersoll 2010

All Rights Reserved

# **The University of Utah Graduate School**

## **STATEMENT OF THESIS APPROVAL**

The thesis of \_\_\_\_\_  
has been approved by the following supervisory committee members:

\_\_\_\_\_**Daniel O. Adams**\_\_\_\_\_, Chair **10/1/2010**

\_\_\_\_\_**K. DeVries**\_\_\_\_\_, Member **10/1/2010**

\_\_\_\_\_**Ken Monson**\_\_\_\_\_, Member **10/1/2010**

and by \_\_\_\_\_, Chair of  
the Department of **Mechanical** \_\_\_\_\_

and by Charles A. Wight, Dean of The Graduate School.

## **ABSTRACT**

Demand for lighter thin-walled tubes have encouraged engineers to use composite materials. However, using composites makes it more difficult to predict material characteristics. Available test equipment is unable to evaluate thin-walled composite tube characteristics. Testing methods and fixtures capable of evaluating tube properties, including flexural strength, flexural fatigue, and flexural damage tolerance, provide a means of comparing product durability. Durability charts (charts with multiple fatigue curves) were produced using test methods from the categories mentioned.

Available testing equipment is unable to evaluate flexural characteristics of thin-walled composite tubes. Available test fixtures are intended primarily for testing flat specimens. Thin-walled composite tubes require unique load applicators to more uniformly distribute loads, thus preventing localized failures. A modified four-point flexural test fixture with rubber load applicators provided satisfactory results.

Flexural fatigue curves were developed by applying various loads, and rotating the tube in a four-point bend configuration. Advantages of this method include having relatively low concentrated loads and a region with a constant bending moment, which is useful when evaluating areas of interest such as joints or impacted regions. This fixture employed the same rubber load applicator concept used on the flexural strength test fixture.

Evaluation of flexural damage tolerance can be done in different ways. Tubes used for this project are visually inspected between each cycle. Thus, short cracks running parallel to the fibers on the outer layer are difficult to detect. An anvil attached to the end of a pendulum seemed to produce damage parallel to tube fibers, similar to that produced during tube operation. The pendulum arc was oriented parallel to the fibers of the outer layer. Damage amounts were determined by tube properties and the amount of tube deformation caused by the impacting anvil.

Composite tube durability charts were produced by developing a fatigue curve using thin-walled composite tubes and two fatigue curves using impacted tubes of different levels of deformation, illustrating the tube's damage tolerance with a vertical shift in different curves. By using the three unique test fixtures developed for this project, it is possible to develop a better understanding of the characteristics exhibited by thin-walled composite tubes.

## CONTENTS

ABSTRACT.....	iii
LIST OF FIGURES.....	vi
CHAPTER	
1 INTRODUCTION .....	1
2 FLEXURE TESTING .....	3
Common flexural strength evaluation fixtures .....	3
Concept modification.....	5
Fixture load point orientation.....	10
Four-point bend fixture validation .....	14
3 FATIGUE TEST FIXTURE .....	17
Fatigue fixture development .....	17
Fatigue fixture validation.....	22
4 PENDULUM IMPACT FIXTURE .....	26
Impact fixture concept development.....	26
Common impact fixture characteristics .....	27
Impact evaluation concepts .....	28
Impact fixture design and production .....	29
Impact fixture operation procedures .....	37
Impact evaluation and fixture validation .....	40
5 CONCLUSION.....	48
APPENDIX.....	51
REFERENCES .....	52

## LIST OF FIGURES

### Figures

1: Shear and moment diagram .....	4
2: Load point assembly .....	7
3: Four-point bend fixture with bearing mounted load points .....	9
4: Central load head .....	10
5: Three tube designs tested using varying outer span .....	14
6: Average maximum bending moment for various tube designs .....	15
7: Fatigue test fixture .....	19
8: Fatigue test fixture loaded with a test specimen .....	20
9: Rubber load point and sleeve assembly .....	21
10: Bending fatigue of several tubes with similar flexural stiffness .....	23
11: Bending fatigue curves for tubes of similar construction with different flexural stiffness .....	24
12: Impact pendulum loads and restraints .....	31
13: Pendulum stress with 440 N in shear and normal forces .....	32
14: Impact fixture's trigger .....	34
15: Pendulum brake activation system .....	36
16: Brake assembly .....	38
17: Impact fixture .....	39
18: A series tube with impact induced crack .....	41

19: Fatigue curve series A, $EI = 6.3 \text{ N m}^2$ .....	42
20: Fatigue curve series D, $EI = 6.3 \text{ N m}^2$ .....	43
21: Fatigue curve series H, $EI = 6.2 \text{ N m}^2$ .....	44
22: Fatigue curve series I, $EI = 3.69 \text{ N m}^2$ .....	45
23: Fatigue curve series J, $EI = 2.55 \text{ N m}^2$ .....	46



# **CHAPTER 1**

## **INTRODUCTION**

Thin-walled composite tubes are used in many applications where loads are applied transversely to the tube axis, which produce bending moments and transverse deformation. Depending on the application, such tubes are made using a variety of materials and fabricated with a variety of diameters and wall thicknesses. To evaluate the performance of thin-walled composite tubes under flexural loading, suitable test methods must be developed with the capability of evaluating tube strength, damage tolerance, and damage resistance.

Compared to traditional engineering metals, composite materials offer increased stiffness and strength properties at a reduced weight. Often, composite tube designs have stiffness and strength requirements which must be achieved while reducing the weight as much as possible. Specific stiffness is the ratio of modulus of elasticity to density. Specific strength is defined as the strength to density ratio. These quantities are important considerations when design requirements necessitate minimizing weight.

The flexural stiffness and strength of tube designs are determined by a combination of material properties, layer orientation, and geometric shapes. Stiffness and strength evaluations can be performed in many ways. Tube properties can be derived using mathematics and basic lamina-level material properties in conjunction with fiber orientation or they can be measured experimentally using the tube as a test article.

Measuring the properties of the thin-walled composite tube as a test article has several advantages. The first reason for this approach is that some strength properties cannot be easily calculated. Second is that tube properties can be measured without knowing lamina-level material properties of the composite material used. Throughout this research project, the focus will be on tube properties, rather than material properties of the composite material.

A product's flexural performance may depend on a variety of factors and material properties. The measure of flexural performance most commonly considered is the ultimate flexural strength. Two less commonly considered measures of flexural performance are the resistance to flexural fatigue failure and its postimpact flexural damage tolerance. The development of testing methods capable of evaluating a tube's ultimate flexural strength, flexural fatigue resistance, and flexural damage tolerance can assist in product characterization and development.

The goals of this research project are to develop test fixtures which can be used to evaluate the three flexural properties described above. Thus, three different types of testing fixtures and methods had to be developed and proven effective for the evaluation of the thin-walled composite tubes used for this research to complete this investigation. While each test method focuses on an individual characteristic, combination of the three is intended to provide improved understanding of tube performance.

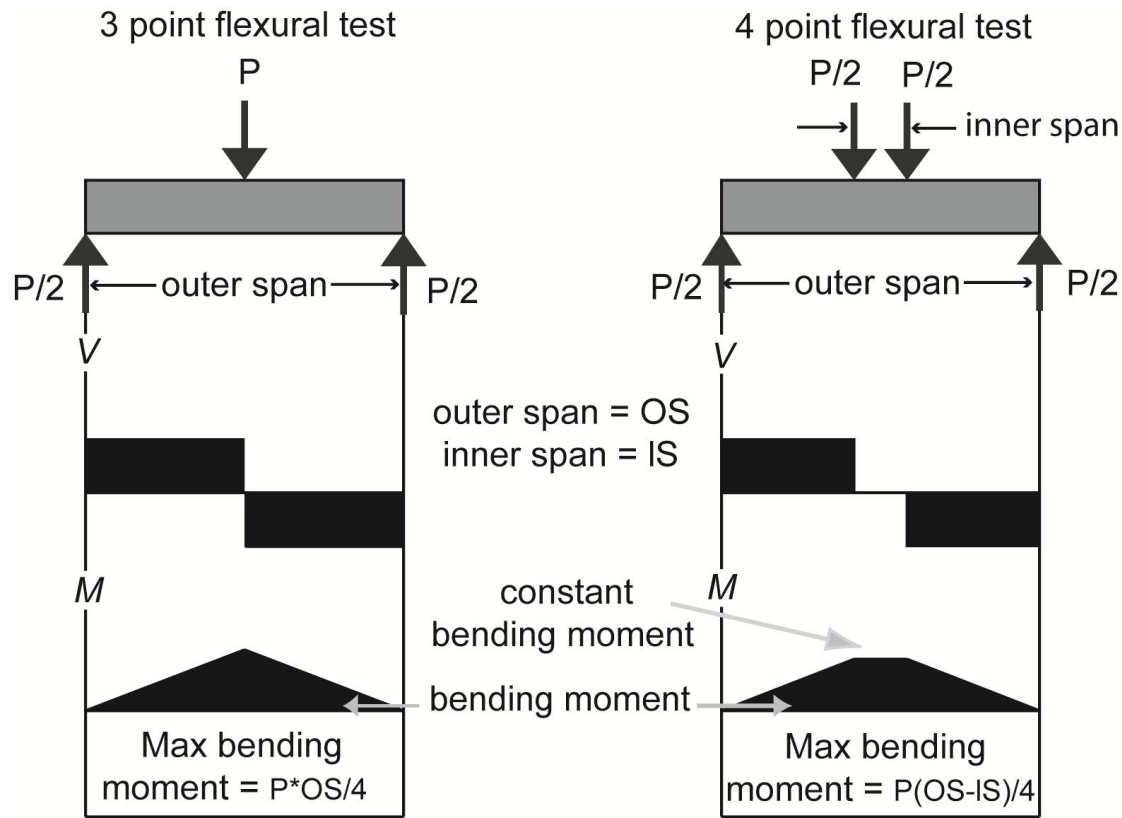
## **CHAPTER 2**

### **FLEXURE TESTING**

#### **Common flexural strength evaluation fixtures**

There are several test methods currently used to evaluate flexural strength. Two of the more common flexural test methods are the three-point and four-point flexural tests (see Figure 1). Three-point flexural tests support the tube on two lower load applicators and apply a transverse load using one inner load applicator. The four-point flexural test supports the tube on two lower load applicators and applies the load with two central or inner load applicators. The load applicators are usually arranged symmetrically about the center of the fixture, as shown in Figure 1.

Each of the above test methods has advantages and disadvantages. The three-point flexural test method creates a larger maximum bending moment, assuming the outer or lower load applicators have the same distance between them and the applied force “P” is the same for each test method. The four-point flexural test method has a region between the two inner or upper load applicators with a constant bending moment. This can be useful for tests where a constant-load "test section" is of interest. The four-point flexural test has an additional advantage in some applications; the force “P” applied is distributed between two upper load applicators. This decreases the applied surface pressure of the load applicators by a factor of two.



**Figure 1: Shear and moment diagram**

Thin-walled composite tube designs are frequently not able to tolerate large shear forces or surface contact pressure. When performing flexural tests, these specimens frequently fail due to one of these factors and not as a result of the maximum flexural load in the concentrated region. The typical locations where such failures are likely to occur is at (or near) the inner load applicators. These failures are caused by a combination of the bending moment and the localized surface pressure applied by the load applicator. Excessive tube deformation under the load applicator can be caused by material failure at the contact point, or ovalization of the tube cross section.

To successfully test the flexural strength of composite tubes, it is necessary to use a test method capable of applying a large bending moment with minimal shear forces and contact pressure. The bending moment can be increased relative to the contact pressure

and shear force by increasing the distance between the outer (lower) load applicator and the nearest inner (upper) load applicator. Additionally, as illustrated in Figure 1, the four-point flexure test distributes the applied forces over two load applicators. This makes it possible to significantly reduce the shear forces and contact pressure for a given applied load "P". The four-point flexural test also has a region between the inner load applicators with a constant bending moment. This is useful for testing specific locations of concern such as joints or impacted regions. As a result of these considerations, the four-point flexure test concept is believed to be the most effective approach for flexural testing of thin-walled composite tubes.

### **Concept modification**

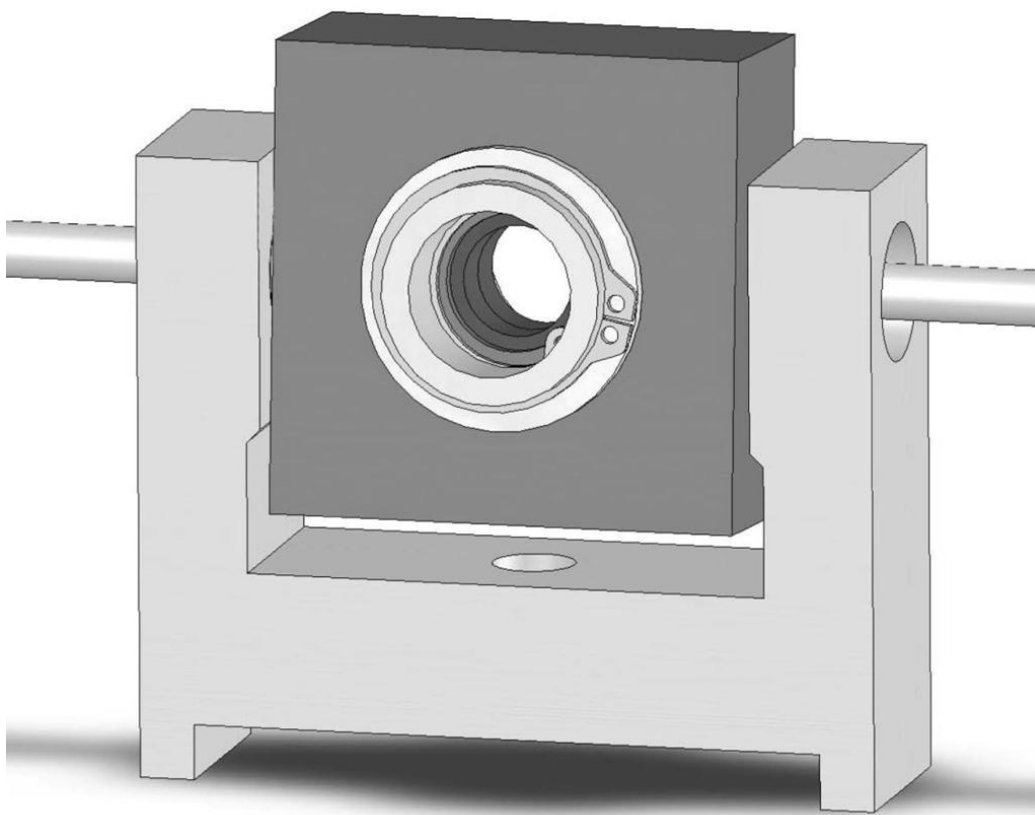
However, additional modifications are necessary to develop the bending moments required. Several modifications to the common four-point flexure tests have been made to produce a fixture capable of testing thin-walled composite tubes.

One of the first modifications necessary is the development of load applicators capable of distributing the load over a larger tube area. Traditional load applicators are cylinders designed for rectangular test specimens, having radii of 6.5 mm. Typically, these load applicators are cylinders made from hardened steel which is mounted perpendicular to the test specimen. The use of these load applicators causes unacceptably large surface pressures when applied to thin-walled composite tube, yet works well on rectangular test specimens because of the larger contact area. The thin-walled composite tubes evaluated in this research project have relatively small outer diameters, below 8 mm. As a result, the use of cylindrical load applicators provides only a very small contact area with which to transfer the load.

To increase the surface area, wider load applicators may be used. A 12.7 mm wide load applicator was utilized in this investigation to reduce the surface pressure. However, the use of wider load applicators introduced new complications. Wider load applicators required the development of a mounting system which allows the load applicator to rotate in the plane of the tube axis, ensuring the load applicator is always positioned parallel to the tube axis at the point of contact. This was accomplished by mounting the load applicator in a block by two 6.3 mm ball bearings. The block is able to rotate, accommodating the changing angles throughout the duration of the test (see Figure 2).

These new load applicators also need to be able to conform to curvatures in two directions: the radius parallel to the tube axis and the radius perpendicular to the tube axis, or the tube radius. The radii parallel to the tube axis needs to be close to the radius of curvature caused by the bending moment. If the radius of curvature induced by the bending moment is smaller than the radius of the load point parallel to the tube axis, the load will be concentrated at the edges of the load-applicator, inducing high surface pressure at the edges. If the radius is too small, the load will be concentrated at the contact point between the load applicator and test specimen, causing high surface pressure. For this reason, the standard four-point flexural test fixture is unacceptable for thin-walled tube testing.

The traditional high-stiffness steels used in the construction of flexural test fixtures are unfavorable for the load applicators. Load applicators need the ability to



**Figure 2: Load point assembly**

conform to the tube specimens, which will have significantly different induced radii of curvature. Such tubes come in different outer diameters, and therefore, the best method of distributing a transverse load requires load applicators be able to conform to the surface of the tube, thus reducing surface pressure during loading.

One material capable of meeting these needs is rubber. Rubber is incompressible yet able to deform to the tube's shape and therefore able to transfer a relatively constant pressure over an extended contact area between the load applicator and tube.

Additionally, rubber can be shaped for a variety of tube diameters and will deform to fit the individual tube curvatures.

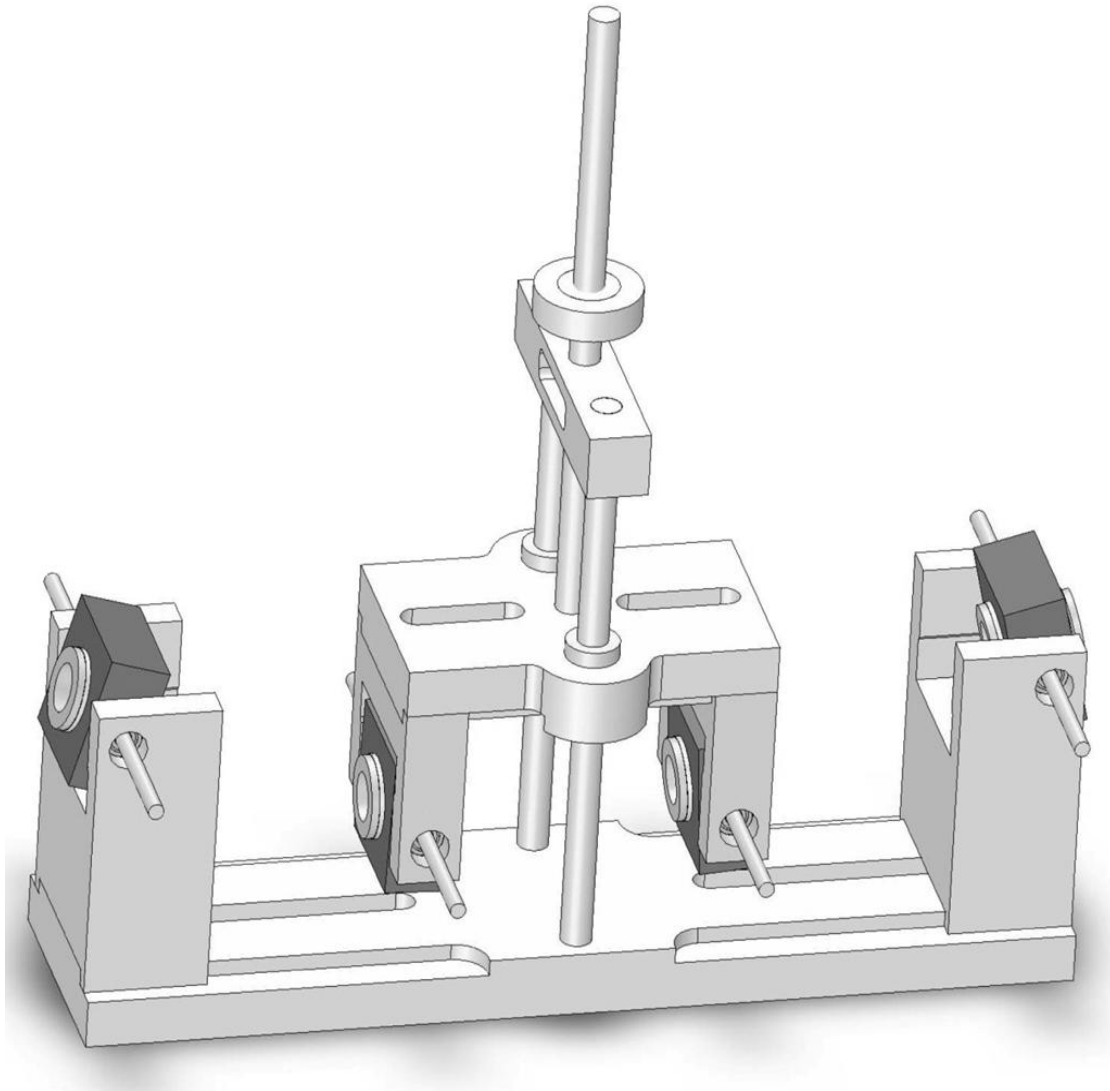
The use of rubber load applicators does introduce limitations. The amount of rubber deformation is dependent upon the outer diameter of the tube, the inner diameter of the load applicator, and the applied force, "P", making tube deflection and stiffness calculations impractical. However, the inability to measure tube stiffness was not a problem for this research project since emphasis was placed on measuring the flexural strength. If tube stiffness was desired, deformation measurements could be made at reduced load levels without the use of rubber load applicators.

With these limitations considered, rubber is still an effective material choice for these load applicators. Rubber load applicators mounted in blocks are able to adjust to the angle of the tube and deform to the contours needed, ensuring relatively uniform pressure distribution. The new load point assembly design illustrated in Figure 2 is capable of transferring the necessary loads without causing localized material failures.

The allowable surface pressure and shear forces are dependent upon the specific tube design being tested. The flexural stiffness of tubes tested as part of this investigation ranged from approximately 2.5 N-m<sup>2</sup> to 8.5 N-m<sup>2</sup>. To accommodate this range of tube designs as well as other tube designs, it was necessary to produce a fixture with an adjustable outer and inner span. The inner span can be adjusted from approximately 6 cm to 13 cm, while the outer span can be adjusted from approximately 20 cm to 40 cm. This means the load point assemblies are able to be set in different locations according to the specific situation or needs. The four-point flexure fixture developed for this project utilized four load point assemblies, as shown in Figure 3.

The ability to relocate the load applicator assemblies provides the ability to test thin-walled composite tubes of different strengths without exceeding the tube's

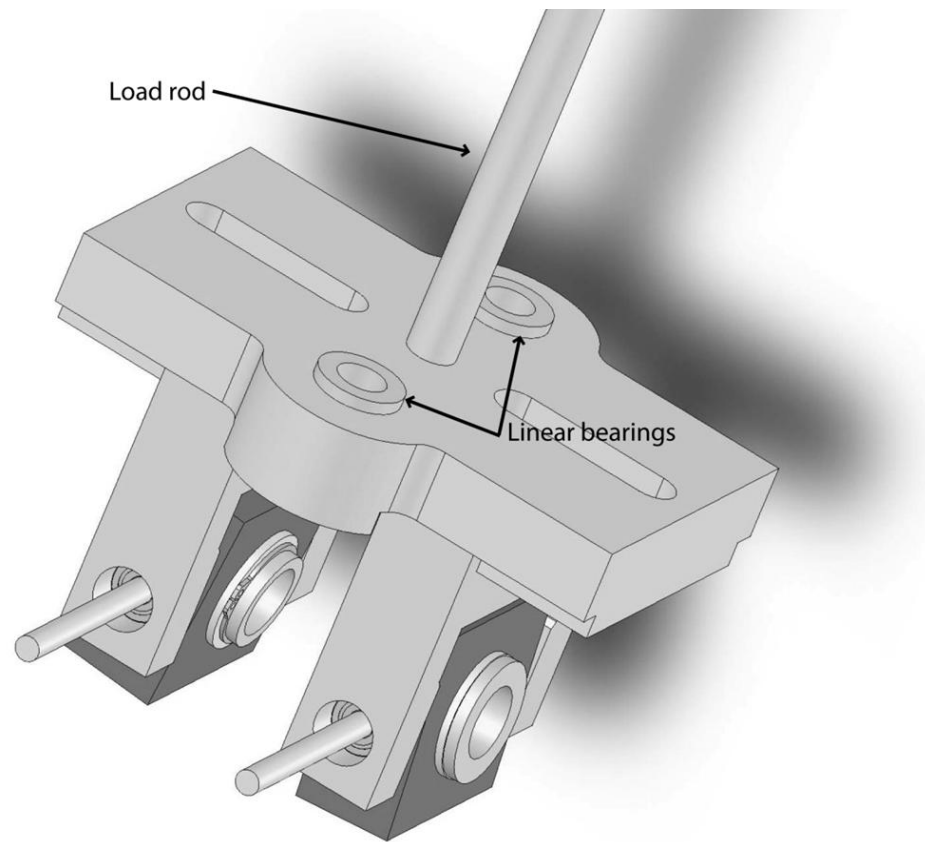




**Figure 3: Four-point bend fixture with bearing mounted load points**

limitations of surface pressure and shear forces. However, this freedom makes it possible to set the load point assemblies in locations which would not be symmetrical, producing undesirable effects because the shear forces and surface pressure would no longer be equal under the corresponding load applicators. Thus, an operator must pay particular attention to the load point assembly locations in order to prevent this situation.

To assist in maintaining a symmetric loading configuration, the fixture is built as a single unit with a central load head (see Figure 4) supporting the two inner load



**Figure 4: Central load head**

applicator assemblies. The central load head is attached to the lower portion of the fixture using two 12.7 mm linear bearings and two vertical procession ground rods. This maintains the load point locations after they have been set, thus preventing the need for constant adjustments.

### **Fixture load point orientation**

If the load point assemblies are set in a symmetrical arrangement, not only will the pressure distribution be the same under each load point, but the equations used to calculate the applied bending moment will be greatly simplified. The bending moment is then a function of three parameters: load, "P", inner span, and outer span (see Equation

1). "P" is the force applied to the central load applicator head (refer to Figure 1). "OS" is the distance between the centers of the outer load applicators. "IS" is the distance between the centers of the inner load applicators. As Equation 1 indicates, increasing the difference between the outer span and inner span has a proportional increase on the applied bending moment, with a given force, "P".

$$M = \frac{P(OS - IS)}{4} \text{ N-m} \quad \text{Equation 1: Applied bending moment}$$

To simplify the operation of the flexural test fixture, either "OS" or "IS" can be set as a constant. There are several things which influence the allowable values for each variable. The fixture limitations on the outer and inner span ranges are only one of the restrictions. The cross head travel is limited by the fixture design as well. The equation for the deflection at the center of the tube under four-point flexural loading may be calculated by using Equation 2.

This equation is a function of four parameters: applied force, "P"; the outer span, "OS"; the inner span, "IS"; and the tube's flexural stiffness, "EI". Deflection is considered as negative because the deformation is downward from the unloaded position.

$$\zeta = \frac{-P}{16EI} \left[ \frac{OS^3}{3} + \frac{IS^3}{6} - \frac{OS * IS^2}{2} \right] \text{ m} \quad \text{Equation 2: Deflection equation [4]}$$

Combining these limitations, the ideal spacing of the load applicators (ideal spans) depend on the tube characteristics. Increasing the inner span increases the deflection required to produce the necessary bending moment, as shown in Equation 2. Thus, the largest bending moment is developed with the smallest inner span, as illustrated

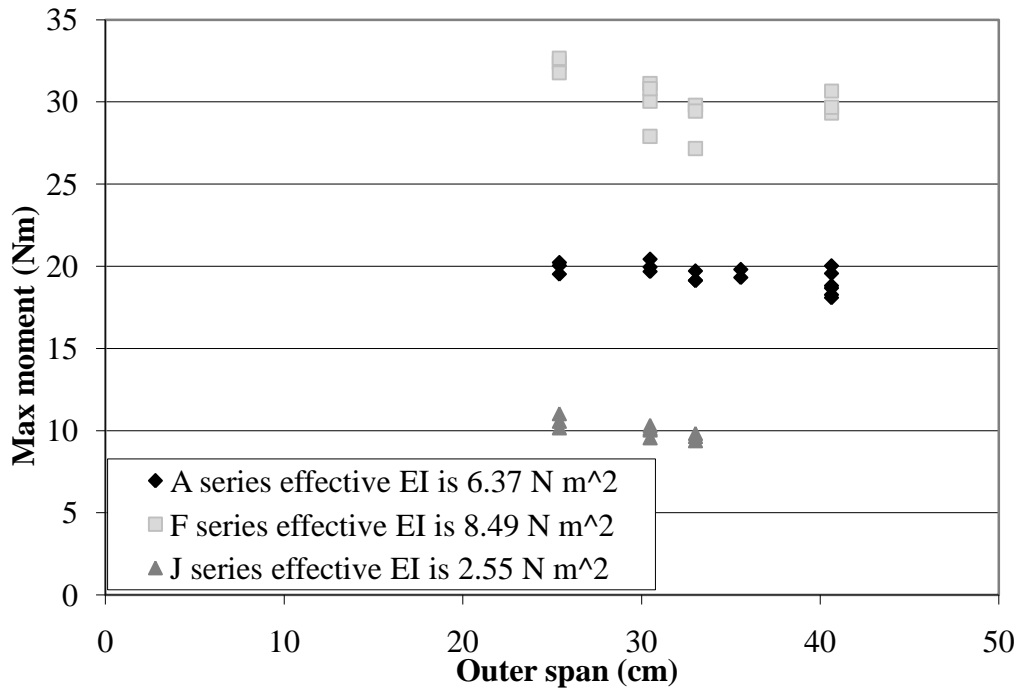
in Figure 1. However, there are several limitations which restrict the minimum span.

The specific test purposes necessitated that a span of approximately six diameters of the tube be subjected to a constant bending moment. The maximum tube diameter which the fixture can accommodate is 8.6 mm. The 6.3 cm minimum spacing allowed by the fixture exceeds this minimum distance. This is also important because the tubes tend to splinter upon test completion, and become difficult to push through the load applicators without damaging them. For this reason, the tested products are removed by pulling them out through the gap between the two central load applicators. Through experimentation, it has been determined that a 6.3 cm span is sufficient for removal of the tested products, making this inner span ideal.

The outer span has the most significant effect on deflection, as can be seen in Equation 2. Increasing the outer span significantly increases the tube deflection and thus the cross-head travel. Different tube designs require different minimum and maximum outer spans. These span limits are determined by the fixture's maximum deflection, the tube's effective stiffness, and ability to withstand surface pressure and shear forces. The parameters which determine the maximum applied pressure or shear pressure are not addressed in this study. Spans below the minimum span for the tube design will be indicated by test results with inaccurate maximum bending moments and the location of test completion will be under or outside the inner load applicators. This is because the tube will not be required to reach its bending limit without the influence of shear forces or surface pressure. Thus, using the minimum inner span for the fixture, each tube has its own maximum and minimum outer span requirements.

The ideal outer span would accommodate all tube designs and was chosen by comparing the minimum and maximum spans experimentally determined for three of the tube designs considered in this study. The tubes chosen for this comparison were the two extremes in flexural stiffness and the most commonly used tube design. These three tubes were tested using a variety of spans. The important point to consider is whether the bending moment is the sole cause of test completion or if it is some combination of the bending moment, surface pressure, and shear force. This can be illustrated by looking for inconsistencies in the maximum bending moment supported by the different tube designs. Their maximum bending moment will be similar inside the acceptable range for the outer span. Any large variation would indicate that the tube is now undergoing some combination of bending moment and shear force or excessive deformation of the tube wall (see Figure 5).

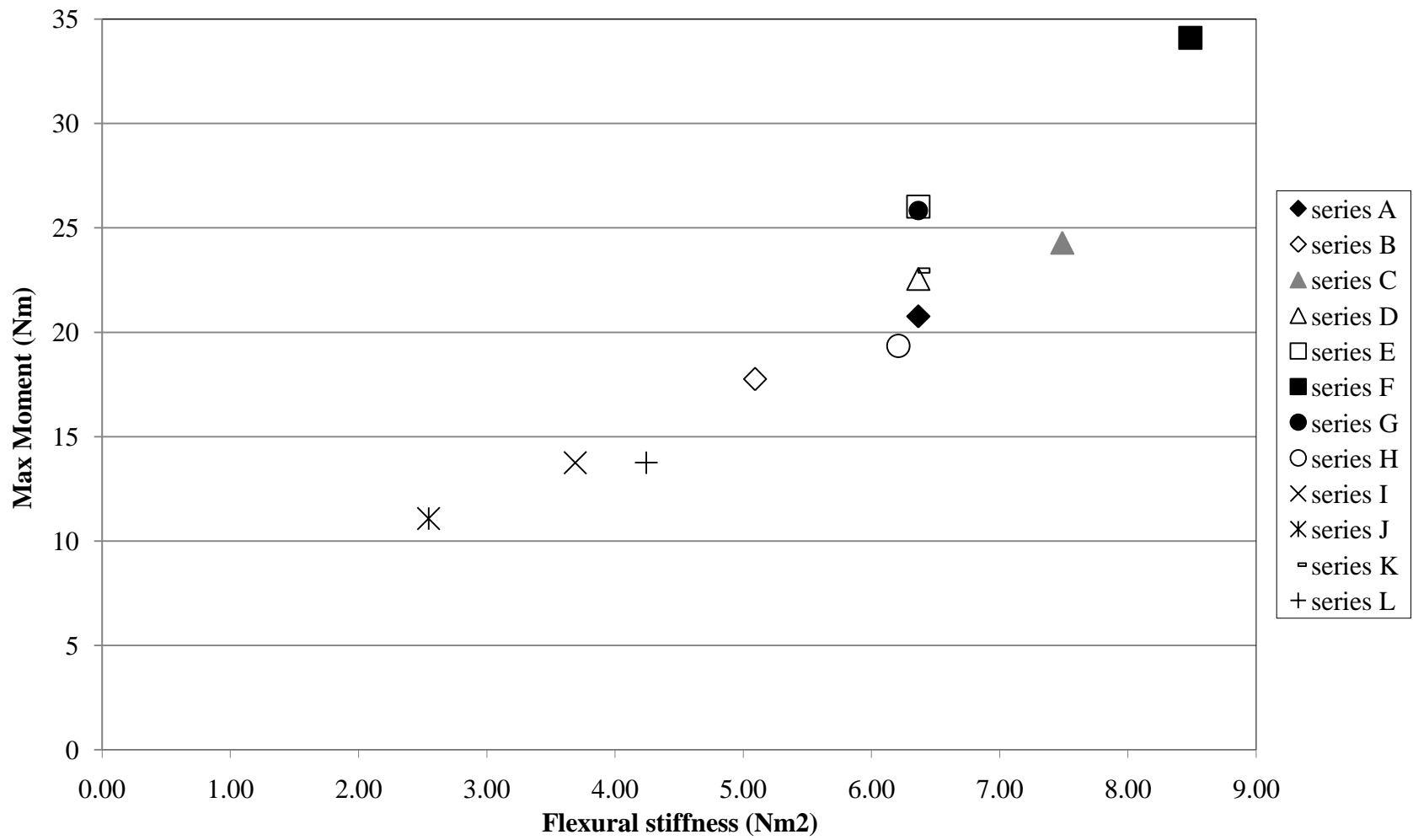
The large jump in the average bending moment supported by the F series tube at a 25 cm span indicates that this span is too small; this theory was supported by crushing damage on the tube walls of these test specimens. Therefore, the tube is undergoing some combination of bending moments, and/or surface pressure. Thus, the minimum outer span for the F series tube must be larger than 25 cm. The maximum outer span is limited by the J series tube design which allows the central load point assemblies to hit the bottom of the fixture if the outer span exceeds 33. The ideal span for the tubes tested, using an inner span of 6.3 cm, is between 30 and 33 cm. The outer span chosen was 30 cm, which is the minimum span capable of preventing other influences (in addition to the bending moment) to modify the test results.



**Figure 5: Three tube designs tested using varying outer span**

#### **Four-point bend fixture validation**

To illustrate the effectiveness of this fixture, twelve different tubes were tested using an outer span of 30 cm and an inner span of 6.3 cm. The maximum bending moment sustained by the tube designs tested is illustrated in Figure 6. The equation used to calculate the maximum bending moment was a simplified form of Equation 1, using the constants which have been chosen. The force is applied on the load rod attached to the central load head (see Figure 4). This applied force is provided by a 5 kip Instron test machine using a cross-head displacement rate of 1 cm/min. The central load head is not designed to be supported by the test machine, which means the 49.1 N gravitational force resulting from the central load head must be added to the applied force, "P", which is recorded by the test machine. Adding this force and substituting the outer and inner span into Equation 1 results in a new simplified bending moment equation, Equation 3.



**Figure 6: Average maximum bending moment for various tube designs**

$$M = \frac{(49.1 + P) * .237}{4} \text{ N-m} \quad \text{Equation 3: Bending moment}$$

Each point in Figure 6 represents the average value of a series of tests performed for each tube design. The standard deviation for any given data point was below 0.9 Nm. This low standard deviation illustrates the consistency of the results obtained using this flexure test fixture to evaluate tubes with an outer diameter below 8.5 mm and an effective stiffness between 2.5 N m<sup>2</sup> and 8.5 N m<sup>2</sup>.

The load applicators were manufactured at the University of Utah. Standard parts such as bearings, rods, and machine screws were purchased. The one major wearing part is the rubber inserts which were designed using 90 durometer neoprene rubber. These inserts will wear and harden over time, requiring replacement.

The rubber inserts were manufactured at the University of Utah using a waterjet cutter. The waterjet cutter has some limitations that must be considered; the two most significant are that it produces a tapered cut line, and the machine is unable to make precision cuts on rubber. The inability to make precision cuts on rubber is partly because the rubber deforms during the cutting process.

In summary, no commercially available standardized test fixture is capable of evaluating the flexural properties of the thin-walled composite tubes used in this research project. The use of this four-point flexural test fixture makes it possible to measure the ultimate flexural strength of such tubes, which are unable to support high surface pressures. This fixture is capable of evaluating flexural strength due to the method of transferring loads to the thin-walled tubes without inducing excessive deformation.



## **CHAPTER 3**

### **FATIGUE TEST FIXTURE**

#### **Fatigue fixture development**

The concepts used in the development of the four-point flexural test fixture, discussed in the previous chapter, can also be implemented in the development of a flexural fatigue test fixture. Fatigue is a process of cyclic loading which causes the material structure to break down [2]. A product's life expectancy is an important aspect of design evaluation which can be better understood with fatigue testing. Actual product life predictions require a test which accurately represents the load conditions experienced in operation. These conditions include spectrum loading (variations in load frequency and pattern) and environmental conditions. Each operation, cycle, or application may involve different environmental conditions and different load spectrums which makes it impractical to attempt all such evaluations.

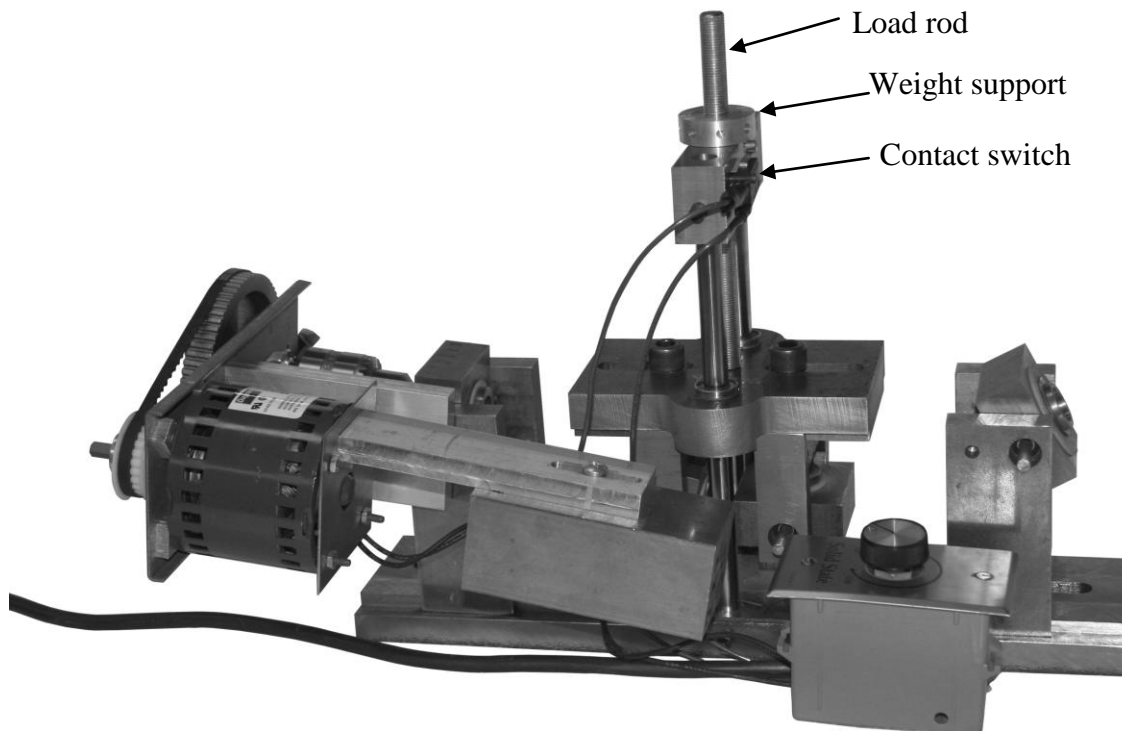
Product durability can be evaluated in a multitude of ways. For this research project, the test must be able to focus on a specific region of the tube. This ability makes it possible to examine the general tube characteristics as well as the effects of damage within the specific region of interest. To make this possible, the fixture needs to have a short span of about six times the tube diameter which is under a constant bending moment. The most reasonable method of achieving this goal is to use the four-point

flexure test concept discussed in the previous chapter. This concept also has the advantage of reduced surface pressure compared to the three-point flexural test, as discussed in the previous chapter (see Figure 1). Incorporating the four-point flexure test configuration would also insure better correlation between the quasi-static flexural tests and the fatigue tests, especially if the spacing between the load applicators remained the same for both fixtures.

There are a few basic methods of performing fatigue tests using the four-point flexural test setup. The first is to incorporate a servo-hydraulic test machine which applies the cyclic loading. With the correct system, it is possible to apply a variety of load spectrums. This would provide for a very versatile test, but is only able to test the tube at one angle orientation. Another method is to rotate the tube while applying a load on the fixture, which makes it possible to test all angular orientations of the tube. The final option is a combination of the two systems, providing maximum control while testing the tube from all angular orientations. This system gives the most freedom in designing the load spectrum, but the control system would be expensive.

Thin-walled composite tubes are axisymmetric, implying that the loading orientation should have no influence on test results. However, it is possible to have slight angular variations when manufacturing thin-walled composite tubes. The possibility of these variations necessitates a testing method with the ability to evaluate the tube at all angle orientations. This is accomplished by rotating the tube while the fixture applies the necessary bending moment.

Free weights were chosen as the method of controlling the applied bending moment. These weights are placed on the load support washer (see Figures 7 and 8).

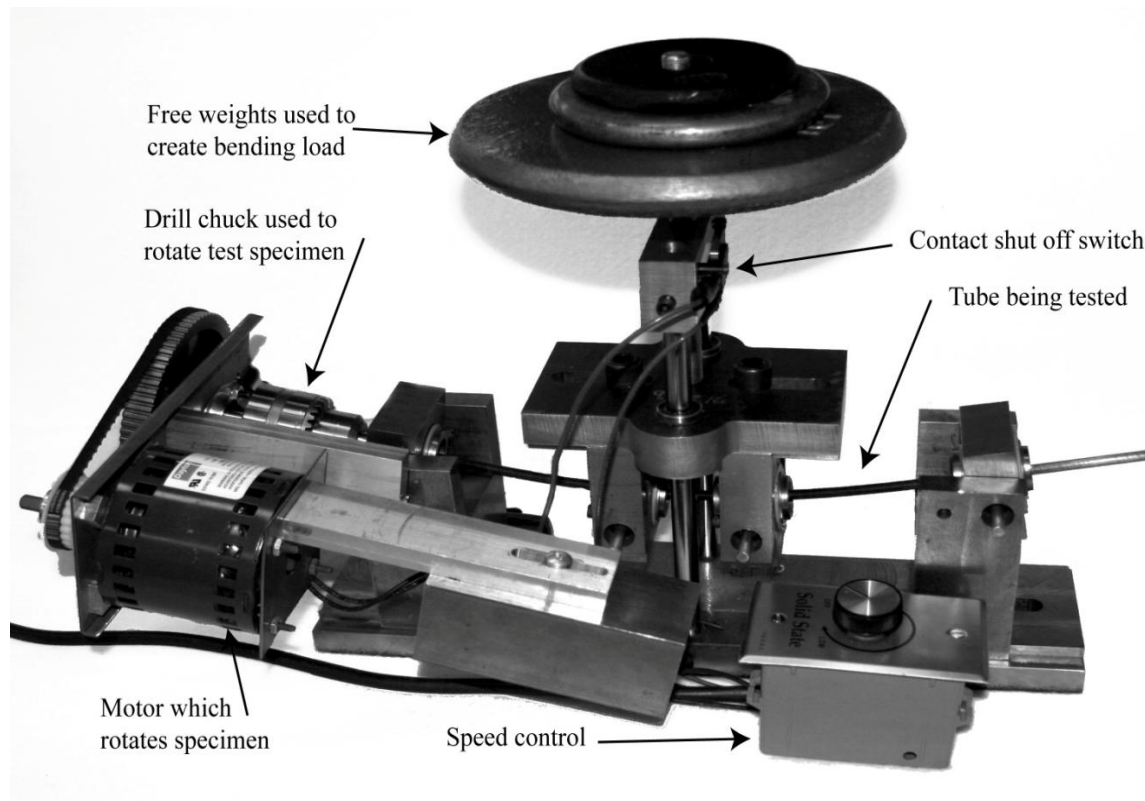


**Figure 7: Fatigue test fixture**

A 1/20<sup>th</sup> hp single phase Dayton 3m547d AC motor attached to a 12.6 mm drill chuck is used to rotate the tube while the cycles are counted by a bicycle odometer.

The motor was chosen because of its size, price, availability, and controllability. It was manufactured to operate a small ventilation fan which received the necessary cooling from the air it moved. This means that the motor needs a fan to provide proper cooling. The motor speed is dependent upon both the load and control setting. A simple wall mount ventilation control switch is used to regulate the motor speed.

It is noted that motors exist which provide better controllability with specific requirements and higher costs. The best options were a three phase motor with a variable



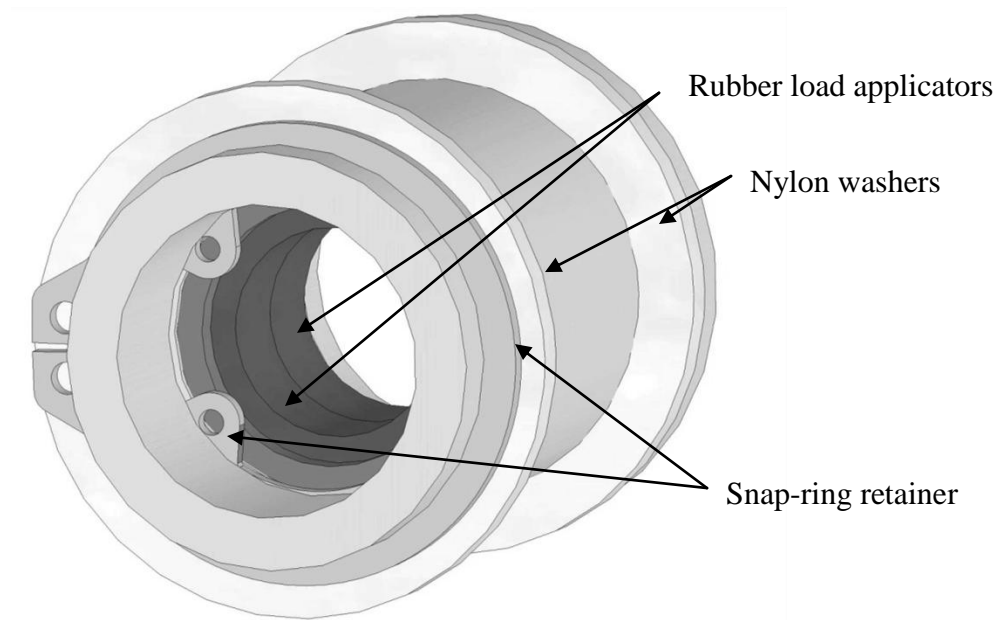
**Figure 8: Fatigue test fixture loaded with a test specimen**

frequency driver (VFD), or a DC motor with the appropriate controller. Each had its own advantages and disadvantages. Three phase motors are not commonly produced in sizes below  $\frac{1}{4}$  hp, and need a VFD to be able to operate at variable speeds. This is an expensive controller capable of maintaining the desired speed independent of load, as long as the load does not exceed the motors torque and power specifications. The DC motors also need special control devices which must be matched to the motor. These motors are made in a variety of available sizes depending upon the intended application.

The AC motor chosen was the least expensive and easiest to acquire. The motor chosen was a  $\frac{1}{20}$ <sup>th</sup> hp motor, which turned out to be smaller than ideal because the load applicator design was modified after the motor purchase. The original load applicators were made with Teflon which provided much less resistance compared to the rubber now

used. A motor size between 1/10 and 1/8 hp would be ideal to operate the fatigue fixture using the rubber load applicators.

Rubber load applicators make it possible to evenly distribute the pressure over a short span of the thin-walled composite tube. The rubber load applicator concept used in the development of the flexural strength test fixture are capable of transferring the necessary transverse load, but lack the ability to accommodate a rotating tube. It became necessary to develop a sleeve mounted in a needle bearing to support two rubber load applicators (see Figure 9). Nylon washers are mounted between the sleeves and bearings to prevent unnecessary friction. The needle bearing is then mounted into a block similar to the one used for the four-point flexure fixture illustrated previously in Figure 2. Snap rings secure the rubber load applicators inside the sleeve. Thus, it is possible to replace the load applicators quickly and efficiently.



**Figure 9: Rubber load point and sleeve assembly**

The load applicators are manufactured using 90 durometer neoprene rubber because of its durability, contact friction between the tube and the applicators, and ability to conform to the tube surface. Each rubber load applicator needs to be about 6.3 mm thick, with an 8.9 mm ID, and a 19 mm OD. Stamping methods are not able to cut the necessary ID through sheet rubber of this thickness, thus necessitating the use of a waterjet cutter. As discussed previously, the waterjet produces a slight taper in the cut surface, and is unable to cut precise shapes in rubber.

The motor is designed to allow for a speed range of 0-1800 rpm, depending upon the control setting and applied load. The load required to drive the flexural fatigue fixture with a 3:1 reduction and the max setting produced a speed of approximately 570 rpm which was used for all the tests performed in this research project.

Each cycle is counted by a bicycle odometer using a magnet attached to the drive pulley. One drawback of using a bicycle odometer to count cycles is that it rounds the number of cycles to the nearest three cycles. This situation occurs because it reports distance traveled in hundredths of a kilometer and counts the rotations as three meters traveled. This is insignificant for tests with a large number of cycles; however, it is a significant limitation when the test is completed in a very low number of revolutions.

### **Fatigue fixture validation**

To illustrate the fixture's ability to evaluate thin-walled composite tubes, the results from five different tube designs with a similar flexural stiffness are shown in Figure 10; and Figure 11 shows results from three different tubes of similar construction methods and different flexural stiffness. Fatigue curves are generally presented with the applied stress vs. number of cycles. This study will use the bending moment versus the

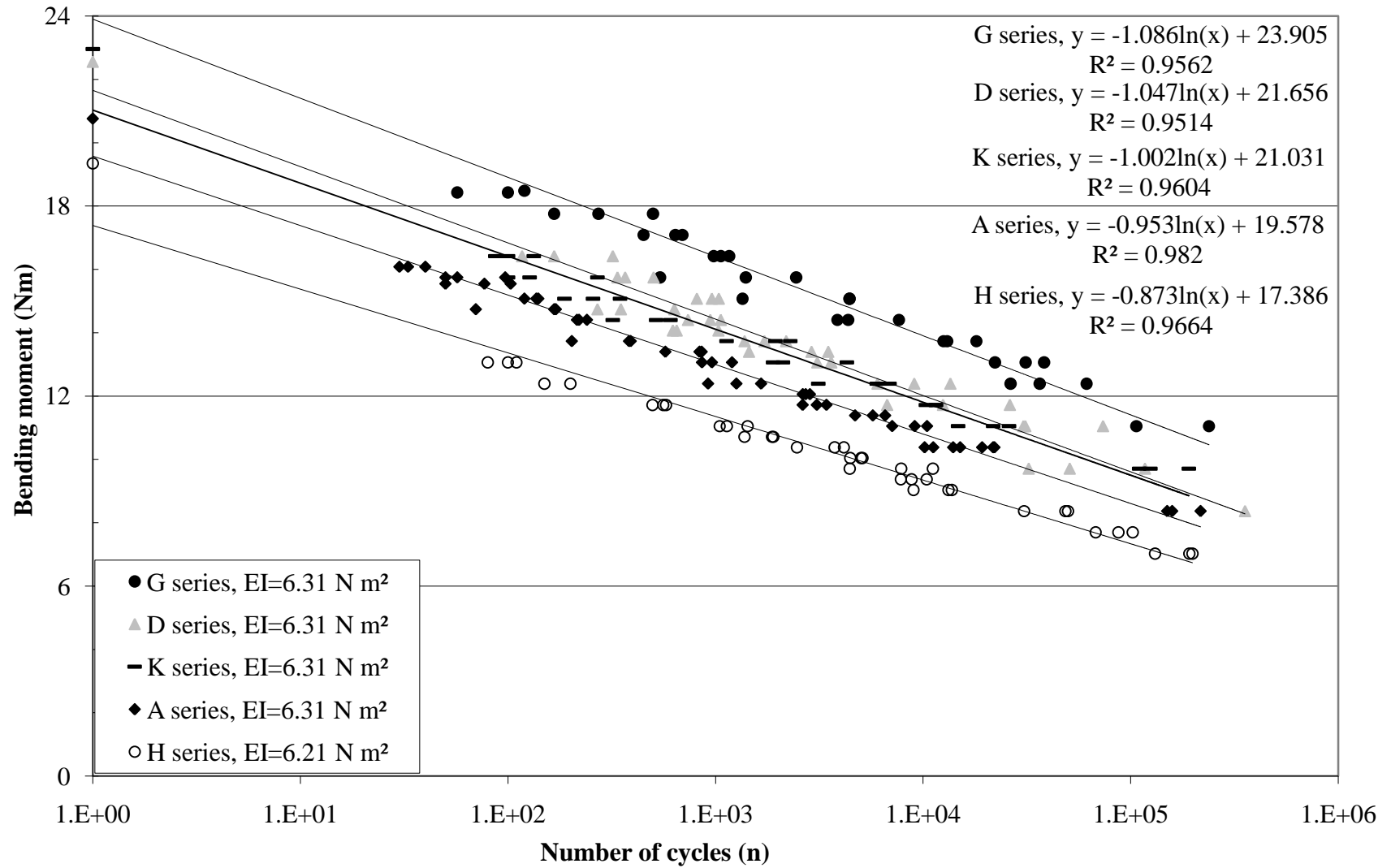
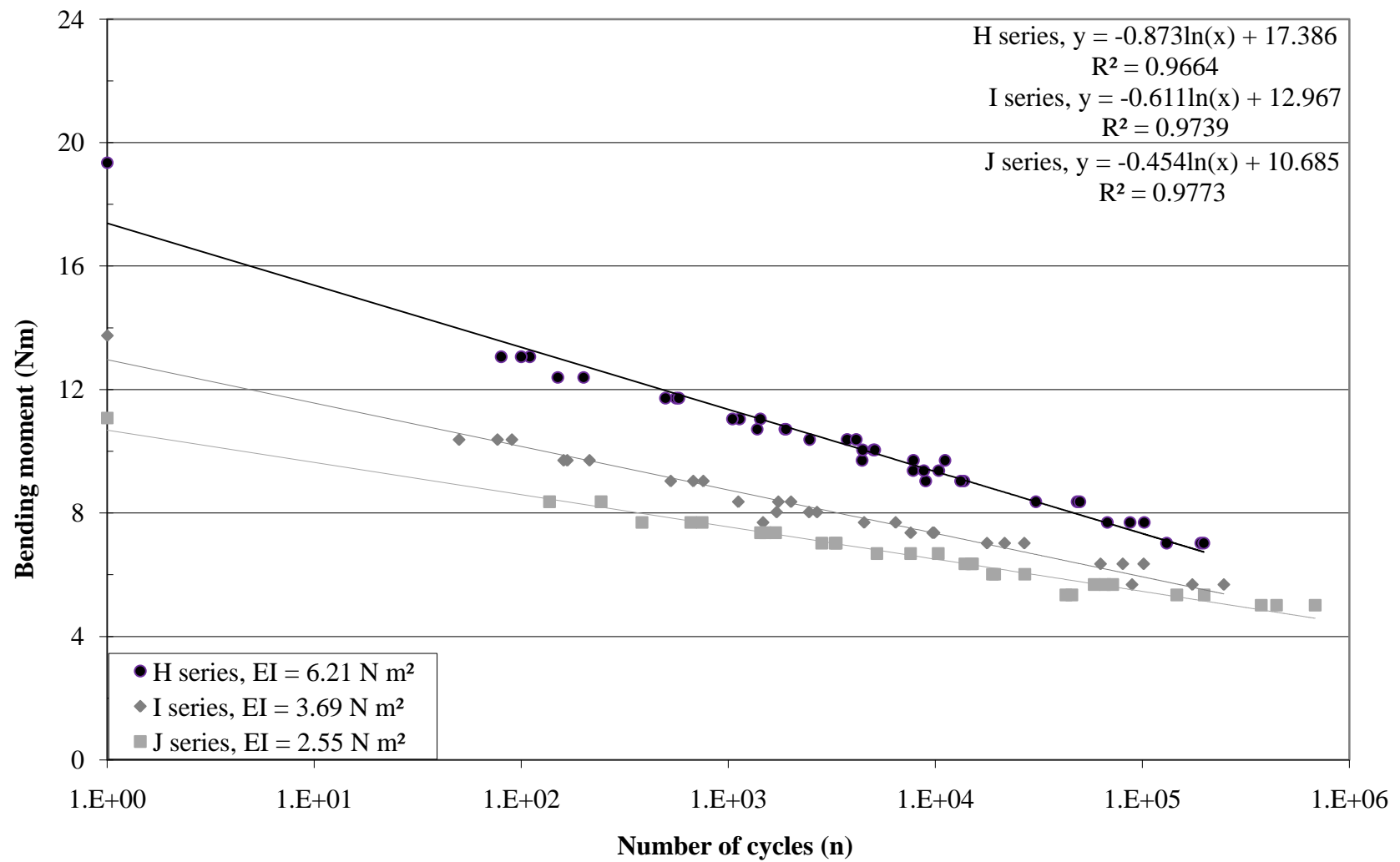


Figure 10: Bending fatigue of several tubes with similar flexural stiffness



**Figure 11: Bending fatigue curves for tubes of similar construction with different flexural stiffness**



number of cycles, making it possible to compare tube designs without knowing layer arrangement or other material properties. The curves generated illustrate tube responses for a series of applied bending moments. Fatigue curves frequently display a logarithmic pattern and therefore, it is common to display the x-axis with a logarithmic scale. This pattern makes it possible to compare the slope generated for different product designs and gain an understanding of their response to actual operating conditions.

Fatigue curves inherently have a large amount of scatter. For this reason it is necessary to repeat each test several times to increase the confidence in results. It is also advantageous to evaluate the effectiveness of the test method by examining the curves using a logarithmic scale on the x-axis which displays the number of cycles. This is because it is easier to see the inconsistencies with a trend line which appears linear.

In summary, the development of a four-point flexural fatigue fixture which rotates the tube made it possible to evaluate the fatigue performance of thin-walled composite tubes. The fatigue curves illustrate the relative fatigue life of different designs, assisting in product development and design. The consistency found in Figure 10 and Figure 11 demonstrate the successfulness of the four-point flexural fatigue test fixture. The fixture is capable of transferring the load necessary to produce the desired bending moments.

## **CHAPTER 4**

### **PENDULUM IMPACT FIXTURE**

#### **Impact fixture concept development**

Damage resistance and damage tolerance are important aspects of product development and design. This investigation focused on the damage tolerance of thin-walled composite tubes. Damage tolerance is the study of how a product will respond to preexisting damage. This means a method of introducing damage is needed before the damage tolerance test can be performed. For this investigation, flexural fatigue testing, as described in the previous chapter, was selected as the type of test to be performed to assess damage tolerance.

There are several methods of introducing the necessary damage. The challenge is deciding what kind of damage can be used to better understand the product, and then to find a quantifiable method of introducing that damage. Tubes are visually inspected between cycles, which means any damage with a high probability of detection through a visual inspection methods does not need to be considered. Forms of damage with a low probability of detection through visual inspection would not include cracks perpendicular to the tube axis. These usually leave a noticeable misalignment or bump in the fibers. Permanent deformations in the form of a dent or gouge are also likely to be noticed

through visual inspections. This leaves short cracks which are parallel to the visible fibers or tube axis.

The goal in developing a damage production fixture is to introduce axial cracks approximately 12 mm to 30 mm long. This can be done several ways, most of which are unlikely in actual use. The most likely thing to cause such cracks during proper use would be impacts with a foreign object. These impacts can occur in many different ways depending upon the object's path before impacting the tube. Possible impacts can be grouped into a few categories: axial impacts, perpendicular impacts, and impacts which occur at some angle with respect to the tube axis. The damage resulting from these impacts depends upon the forces involved and relative angle between the tube axis and object path.

### **Common impact fixture characteristics**

Impact damage tests are frequently performed on composite materials using an impactor with an instrumented impacting head. When the head strikes perpendicular to the test specimen, dents or fiber cracks are generally produced, which are easily detected through visual inspection methods. Thus, this test method does not produce the kind of damage seen in operation and of interest to product development teams.

Most impacts which cause unnoticeable damage strike the tube's outer wall and drag along the side, parallel to the tube axis, for some distance before pressure is removed. These impacts will be referred to as glancing impacts. Glancing impacts can produce impact force versus time curves which appear similar to those produced by perpendicular impacts. However, there are additional influences which can cause spikes or inconsistencies in the force curves generated by glancing impacts. These spikes can be

caused by vibrations in the impacting head or anvil surface, variations of friction between the tube and anvil, actual damage sustained by the tube, and other unknown factors.

Additional tests are necessary to understand the relationship between discontinuities in the force versus time curves and damage resulting from the impact.

Evaluating the effects of impact damage requires additional testing. Fatigue testing is one method of evaluating the damage tolerance. By comparing the fatigue curves created using impacted tubes to the fatigue curve generated with nonimpacted tubes, the damage tolerance can be assessed.

### **Impact evaluation concepts**

To effectively compare the results from fatigue tests performed on impacted tubes, it is necessary to make some generalizations about the impact. These generalizations will be used to categorize the impacts, eliminating the necessity of investigating the inner layers for delamination or any other discontinuities. There are five major factors which influence tube damage: the amount of deformation the tube is subjected to during the impact, the impact orientation, the nature of the impact, tube properties, and the maximum reaction force registered by the anvil or impacting head. The relationship between these criteria is unclear; each tube has a critical level of deformation at which point increased deformation does not influence the reaction force; rather, it permanently deforms the tube wall. Thus, reaction forces are the most useful criteria for categorizing tube impacts which cause damage with a low probability of detection through visual inspection.

Developing a methodology capable of using the reaction force to categorize the glancing impacts is impractical. The natural choice for impact control is to specify the

tube deformation during impact, because it directly influences reactions forces. Using tube deformation to control reaction forces requires that several tests be performed to gain an understand of each tube's reaction to deformation.

### **Impact fixture design and production**

Thus, an instrumented impacter must be developed to strike the tube surface parallel to the tube axis of symmetry. The impacter must have a system which allows the operator to set the amount of tube deformation and impact energy. To provide a glancing blow, it is necessary for the impacting anvil to follow a path which gradually deforms the tube to a maximum tube deformation and then gradually reduces tube deformation. The easiest method of providing the impact described is to attach the anvil to the end of a pendulum. This will increase the deformation until the pendulum is perpendicular to the tube and then decrease the deformation at the same rate. The maximum tube deformation can be controlled by adjusting the distance between the tube and the center of the pendulum arc. The use of a pendulum impact fixture provides the desired impact.

It is important to insure the pendulum does not impact the tube a second time. Preventing a second impact requires the fixture to include a system capable of absorbing the excess energy and preventing the pendulum from moving back towards the tube after it has rebounded.

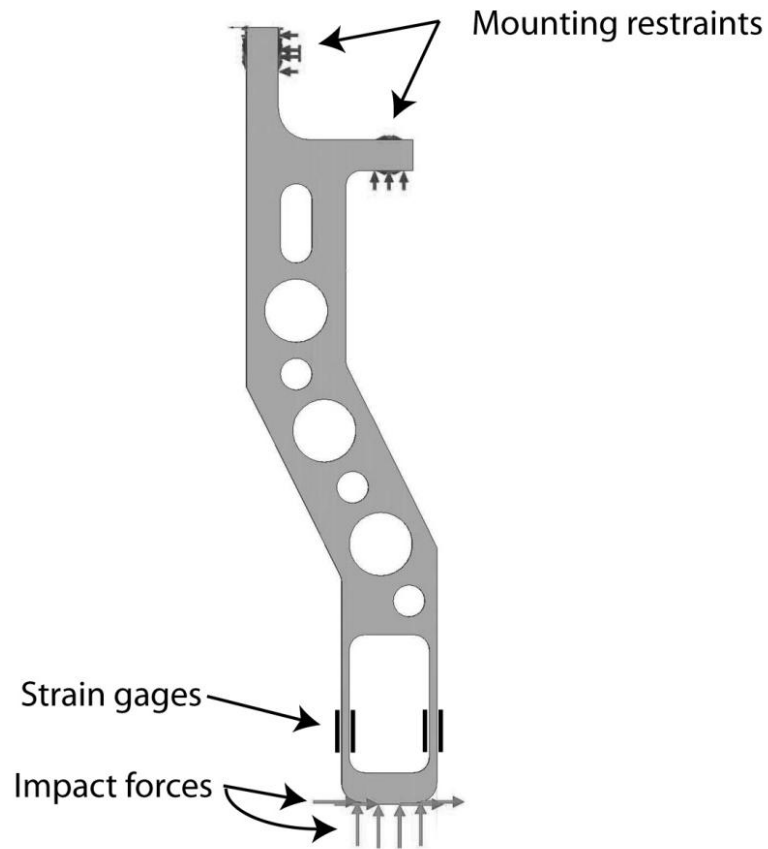
The design of this glancing impact fixture can be broken down into five different major sections. The basic pendulum concept is based on a rotating pendulum supported by a frame. The sections which will be discussed in detail are the pendulum with load measuring capabilities, the trigger release, potential energy used to power the pendulum,

the system which stops the pendulum after impact, and the tube deformation adjustment system.

The pendulum must be capable of measuring the resulting forces in two directions. The most suitable method of measuring these forces was to design an area of the pendulum to elastically deform under a given load and use strain gages to measure the deformation of that area. Several different geometric shapes were considered for the load measuring device. The ideal design must have minimal deformation under the given loads, while undergoing sufficient strain to make it easy to measure the load. Each design had one direction that produced bending moments which have significantly larger strains than strains produced by normal stress with an equal force. To decrease the bending strain without having to reduce the applied forces, it was necessary to position the strain gages as close to the impacting anvil as possible (see Figure 12).

The load measuring system is comprised of four strain gages positioned upon two vertical sections of the pendulum spaced 1.9 cm apart. The strain gages are located as close to the anvil as possible. They must be slightly above the inner radius of the gage area to prevent the stress concentrations and below the center of the measuring area to detect bending loads.

The use of four strain gages makes it possible to decouple the forces in the two directions. The induced normal strain is calculated as the average strain of all four strain gages. The strain follows the general equations used for calculating normal strain (see Equation 4). The forces parallel to the tube axis, perpendicular to the pendulum, are more complicated. The pendulum is forced to bend in the form of an S (see Figure 13). This means the gages will experience different strains. Looking at the gages in



**Figure 12: Impact pendulum loads and restraints**

Figure 12 from left to right, the induced bending creates a compressive stress on the first and third strain gages, and tensile strain on the second and fourth strain gages. Focusing upon this relationship, it is possible to single out the bending stress from the normal stress.

$$\epsilon = \frac{F_n}{E * Area} \quad \text{m/m} \quad \text{Equation 4: Normal strain}$$

Figure 13 depicts the induced stress from an assumed impact on the pendulum. The simulation assumed the reaction forces would be at a maximum of 440 N in both

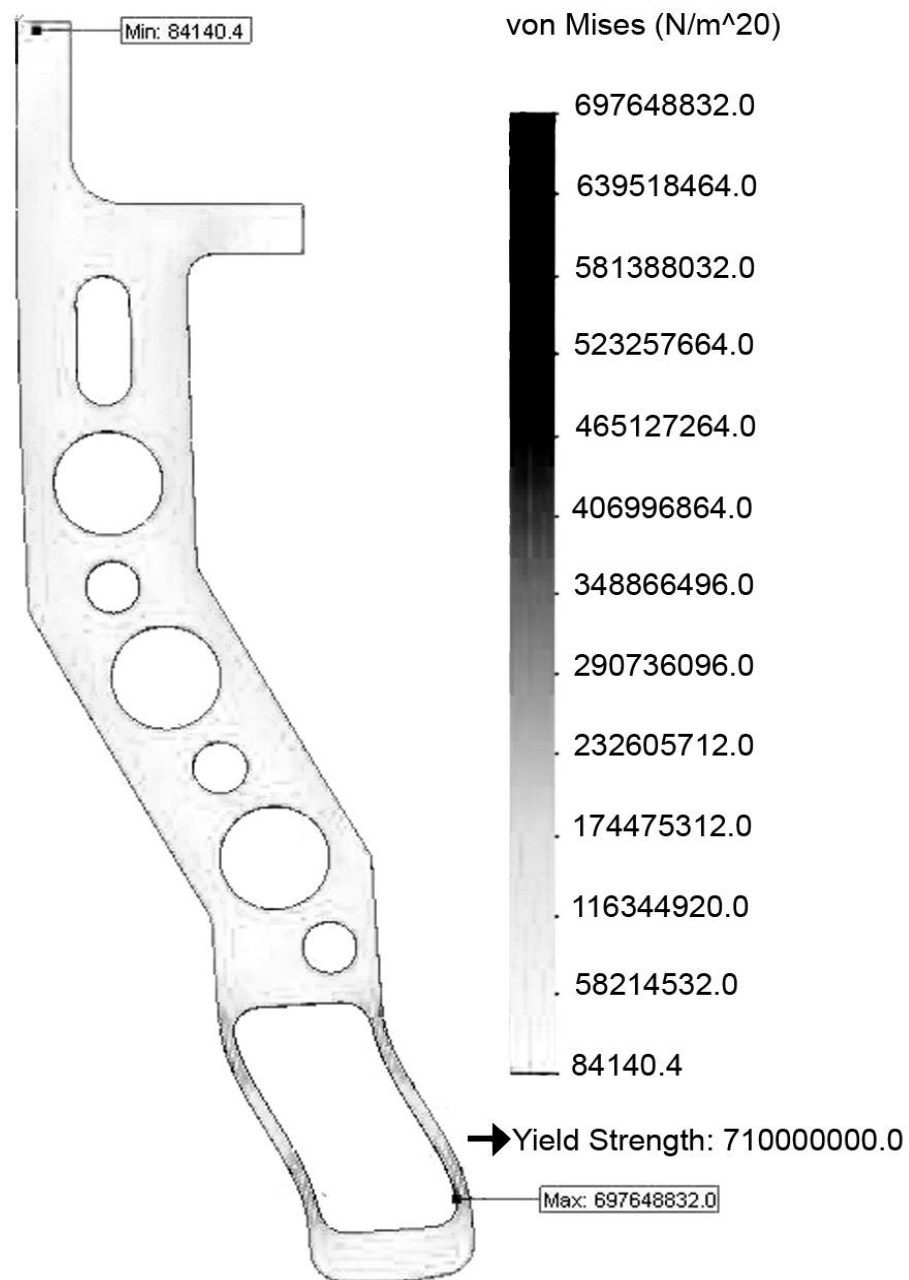
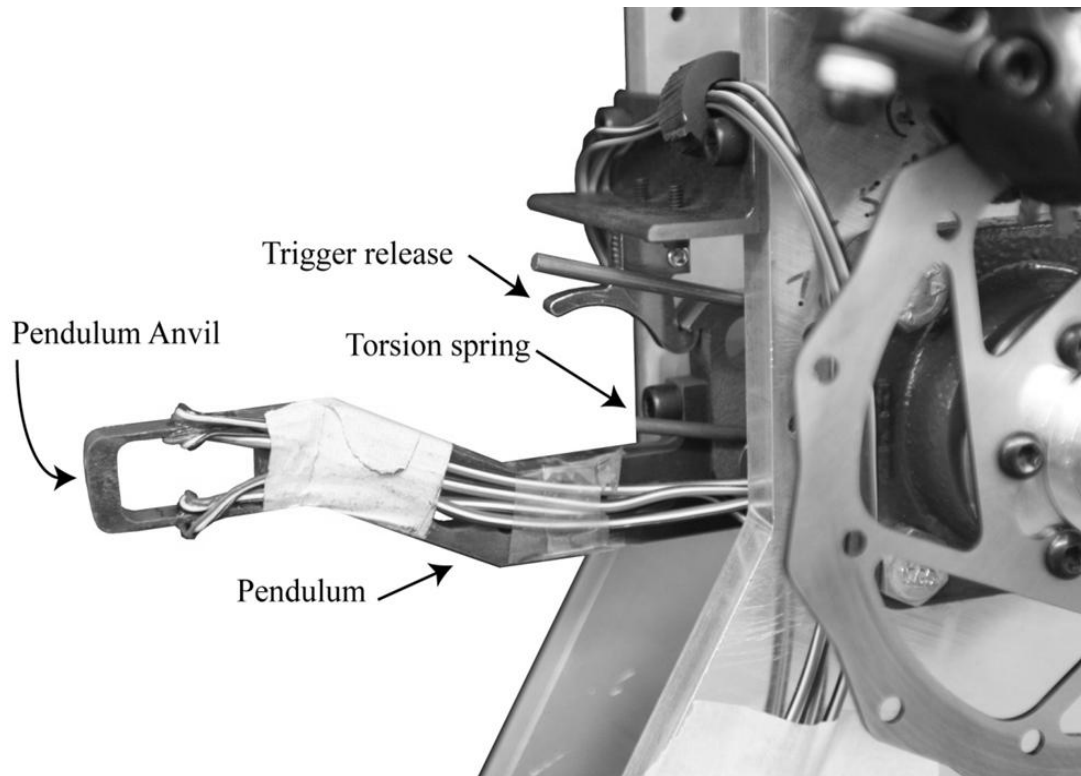


Figure 13: Pendulum stress with 440 N in shear and normal forces



directions (see Figure 12). COSMOSExpress was used to perform finite element simulations. The material properties used for the analyses were those of normalized 4340 steel with a yield strength of 710 GPa, and a Rockwell hardness of 35 on the C scale [3]. The material used for the pendulum is 4340 steel heat treated to a Rockwell hardness of 60 on the C scale, though the hardness of the finished product may vary because the material was machined after heat treating. The possible variations in yield strength make it important to consider all calculations in areas which have been machined to be similar to normalized 4340 steel rather than the heat treated material used.

Heat treated 4340 steel was used because of its resistance to wear. The anvil is subject to wear as it strikes the abrasive composite tube walls. This made it necessary to design the pendulum using heat treated 4340 steel with a Rockwell hardness of 60 on the C scale. Unfortunately, it is impossible to heat treat the material after the shape is cut because the gage area will undergo significant and undesirable deformation in the process. The raw material needed to be treated before the pendulum shape is cut; however, it is difficult to cut materials of this hardness. This pendulum is mounted to a horizontal shaft in the impact fixture frame by two bearing blocks from McMasterCarr, part number 5967K31. The rotational energy needed to power the impact pendulum is provided by a torsion spring capable of developing 4.8 N-m torque, also from McMasterCarr, part number 9271K118. Pulling the pendulum into the spring-loaded position applies the necessary mechanical energy in the spring to create the desired impact. A spring-loaded latch from McMasterCarr, part number 11265A71, prevents the pendulum from releasing before the operator is ready (see Figure 14).



**Figure 14: Impact fixture's trigger**

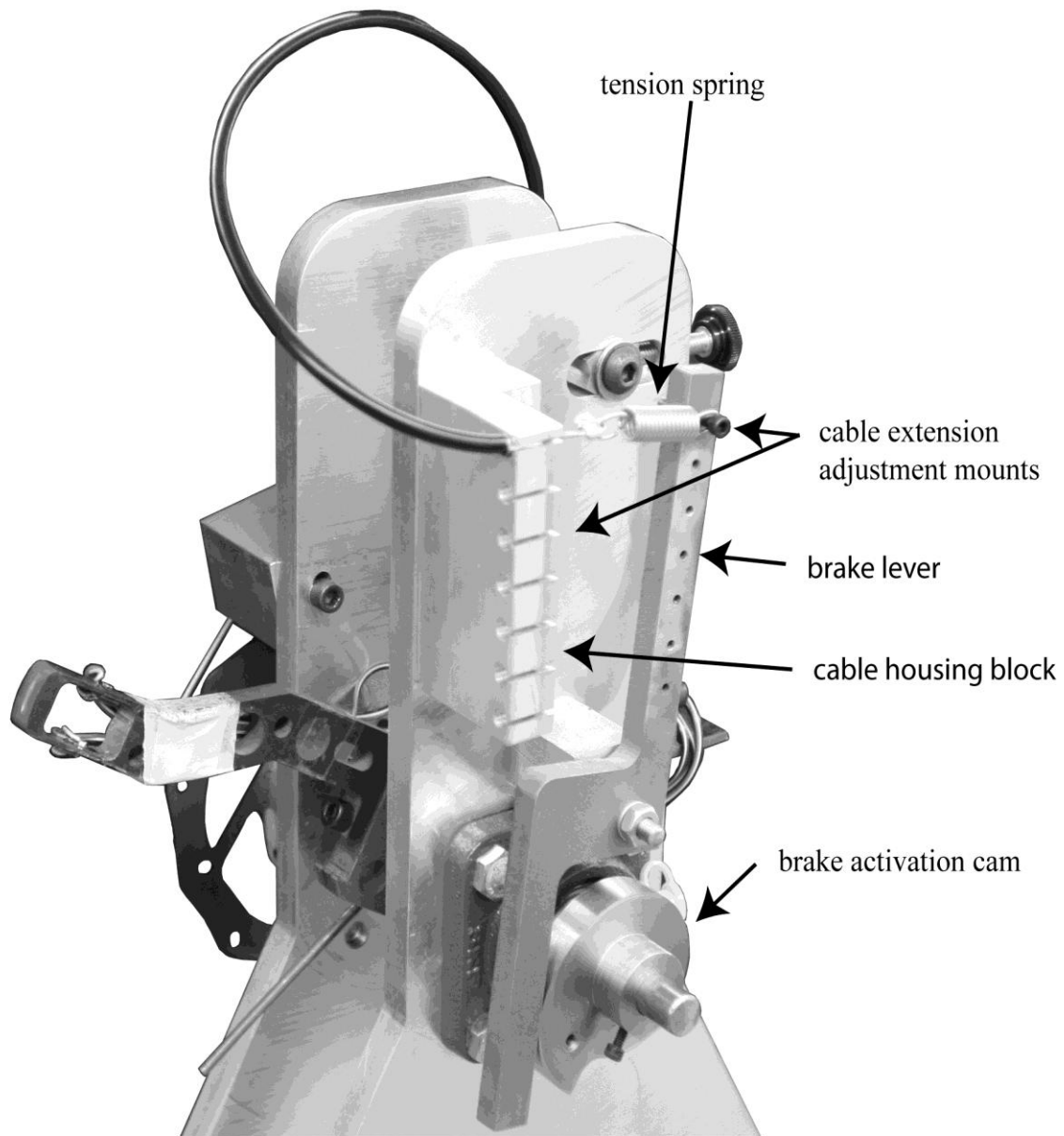
Upon release of the pendulum, the spring provides sufficient energy for the pendulum to damage the tube and continue in its rotation. Unfortunately, the energy required to perform this task is capable of damaging the pendulum's load measuring system if the rotation is stopped abruptly. This occurs when the pendulum is allowed to impact an immovable object such as the impact fixture's frame. It is equally important to prevent the pendulum from impacting the tube more than once because it could increase the damage. To prevent the pendulum from impacting the framework or the tube multiple times, it is necessary to stop the pendulum after the impact.

The best method of stopping the pendulum in this application is to incorporate a brake system. The system chosen includes a disk caliper, model bb7-mtn, and a 140 mm disk made by Avis. This combination was chosen for several reasons. The caliper was

chosen because it is cable operated, and the 140 mm disk is the smallest disk commonly available for use with the caliper chosen.

As the pendulum rotates, it passes the trough used to secure the tube, and the brake activation cam begins to engage the brake system. As the cam rotates, it separates the brake lever and the cable housing block. The movement of the brake lever stretches the tension spring, providing the necessary tension to activate the brake caliber (see Figure 15). The braking power can be adjusted by either changing the spring or the cable extension adjustment mounts. The change in the distance between the cable mounting point and the cable housing is determined by the cable extension adjustment mount and corresponding cable housing mount hole used; the ones closest to the pivot point have the least travel and therefore least pressure on the spring.

The pendulum is mounted at a fixed location in the fixture. This means that in order to be able to control the amount of damage induced by an impact, it is necessary to have a system which gives the ability to adjust the location of the tube. Unfortunately, this task is more complicated than it would appear. The ideal method would reference the tube surface closest to the center of the pendulum arc. This would make it easy to categorize and control the amount of tube deflection with minimal adjustments and measurements, even with the different tube diameters. The problem with this design is that it would require a specific span of the tube to have insufficient support. The deflection of the tube support or trough allowed by the unsupported span would exceed the desired deflection of the tube. The only method of eliminating this deflection is to eliminate the unsupported area under the trough.



**Figure 15: Pendulum brake activation system**

The method finally chosen to adjust the trough position uses shims under the trough. This provides a solid support preventing deflection and the ability to provide small position adjustments. The trough is clamped to the fixture after the adjustment shims are added which insures the position will remain constant while many tests of the same parameters are performed.

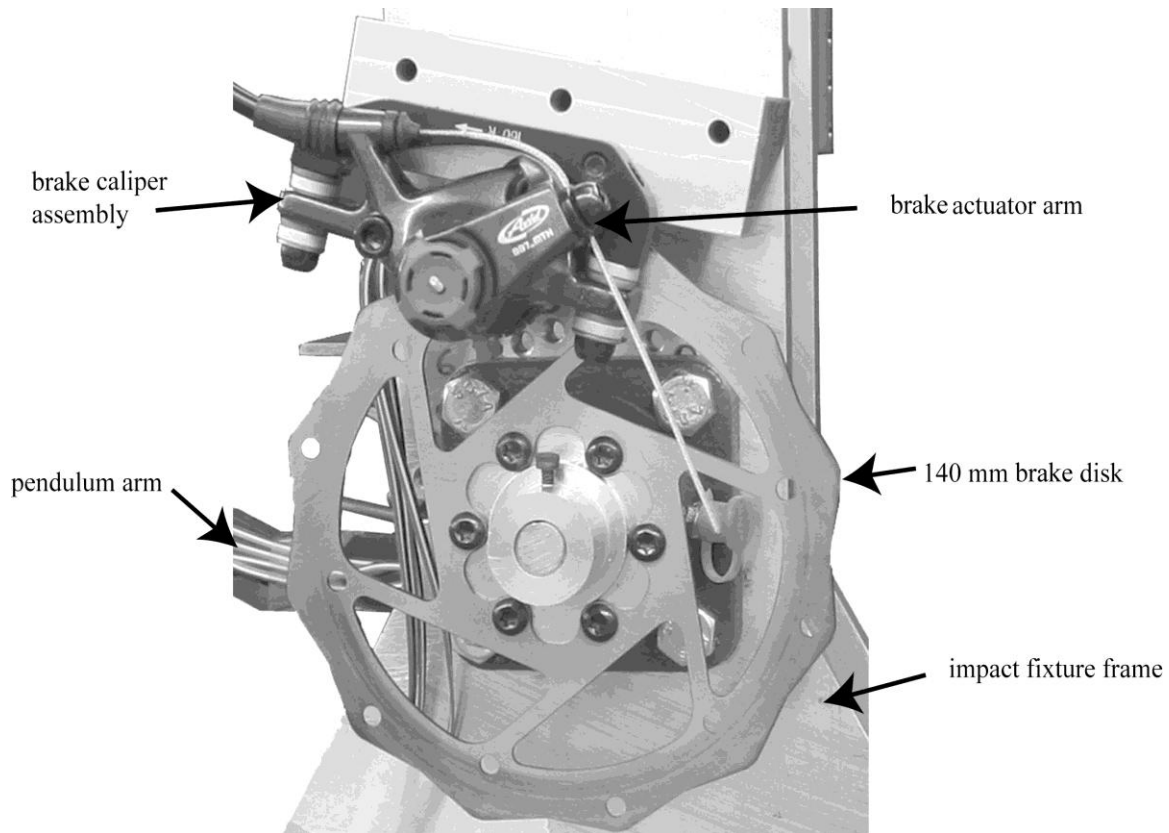
This method has its limitations which include difficult to specify the exact deformation caused by the shims, multiple shims are frequently required, and the reference position is located at the bottom of the tube surface. The trough must be adjusted to maintain the same amount of deformation when testing tubes of different diameters. It is also important to keep the shims clean or the desired and actual deformations will not correspond.

The difficulty of achieving a precise level of deformation is not a critical problem, because the tests focus on a set normal force which requires trial and error testing before the necessary deformation is known. The procedure required to find the desired tube deflection must be done in the appropriate order for the effectiveness and safety of the operator.

### **Impact fixture operation procedures**

The first step in preparing to test a tube is to remove the brake pressure, by twisting the brake actuator arm (see Figure 16) from the system, allowing the pendulum to naturally swing down to the rest position. The operator will need to overcome the tension in the tension spring to perform this task (see Figure 15). Once the pendulum is in the natural position, the clamps can be removed from the tube support trough, making it possible to remove or add shims as needed for the tube series being evaluated (see Figure 17).

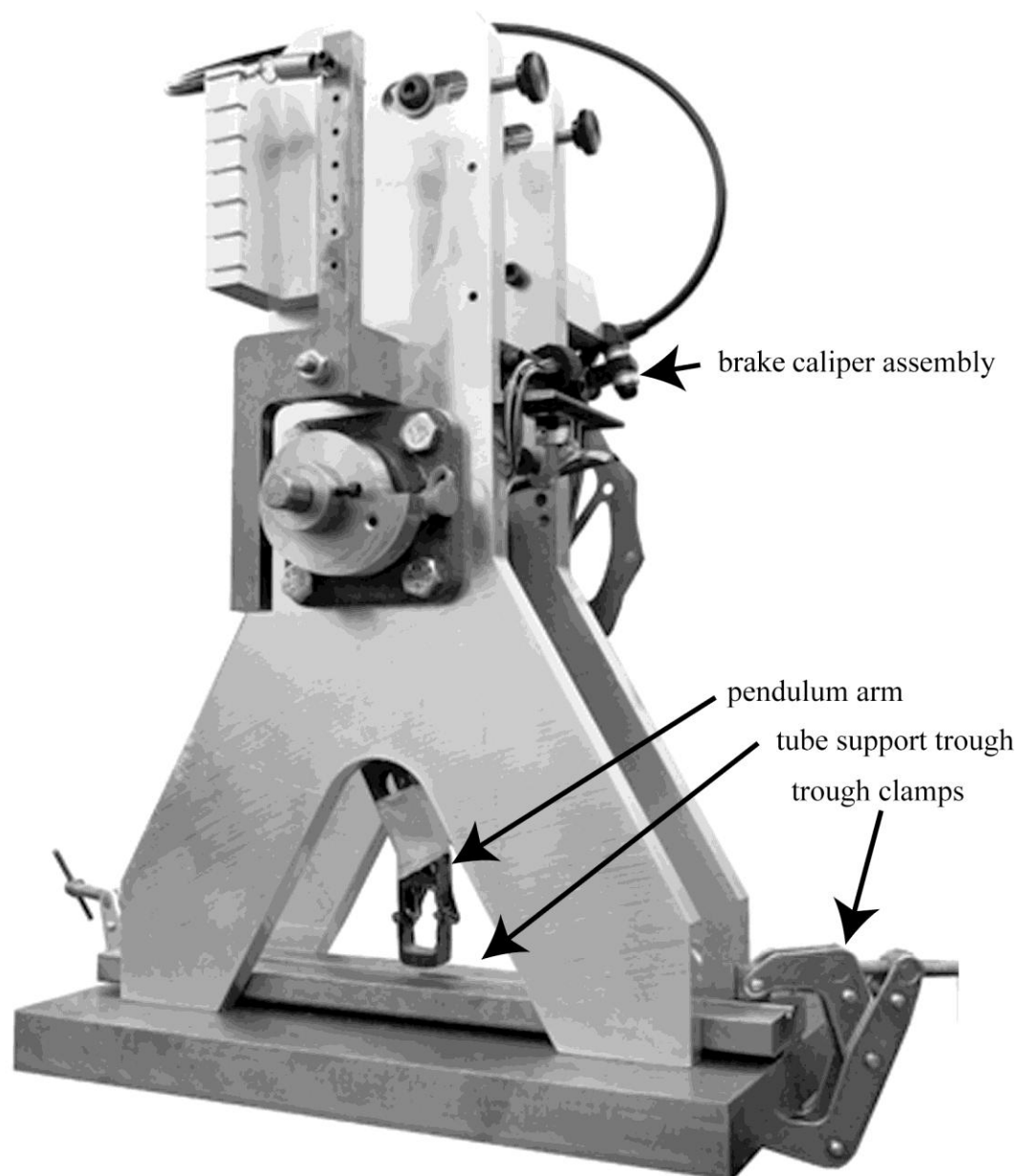
After the tube support trough is set to the desired height, the pendulum is pulled towards, but not into, the spring-loaded position to allow the tube to be slid into place



**Figure 16: Brake assembly**

without interference from the pendulum anvil. Two additional clamps are used to prevent the tube from moving, thus ensuring the damage is located in the desired area. The pendulum is then spring-loaded and ready to impact the tube.

Several tests are required to find the amount of tube deflection, and shims needed to produce the desired reaction forces. To reduce the effects of possible manufacturing variations in the tube series being tested, it is better to use more than one tube while finding the necessary shim arrangement to achieve the desired reaction forces. It is also necessary to perform several tests at the desired tube deflection to ensure the reaction force is not an outlier in the scatter present in this form of testing. Once the optimal tube deflection is found, all the reaction forces are averaged for ease of data interpretation.

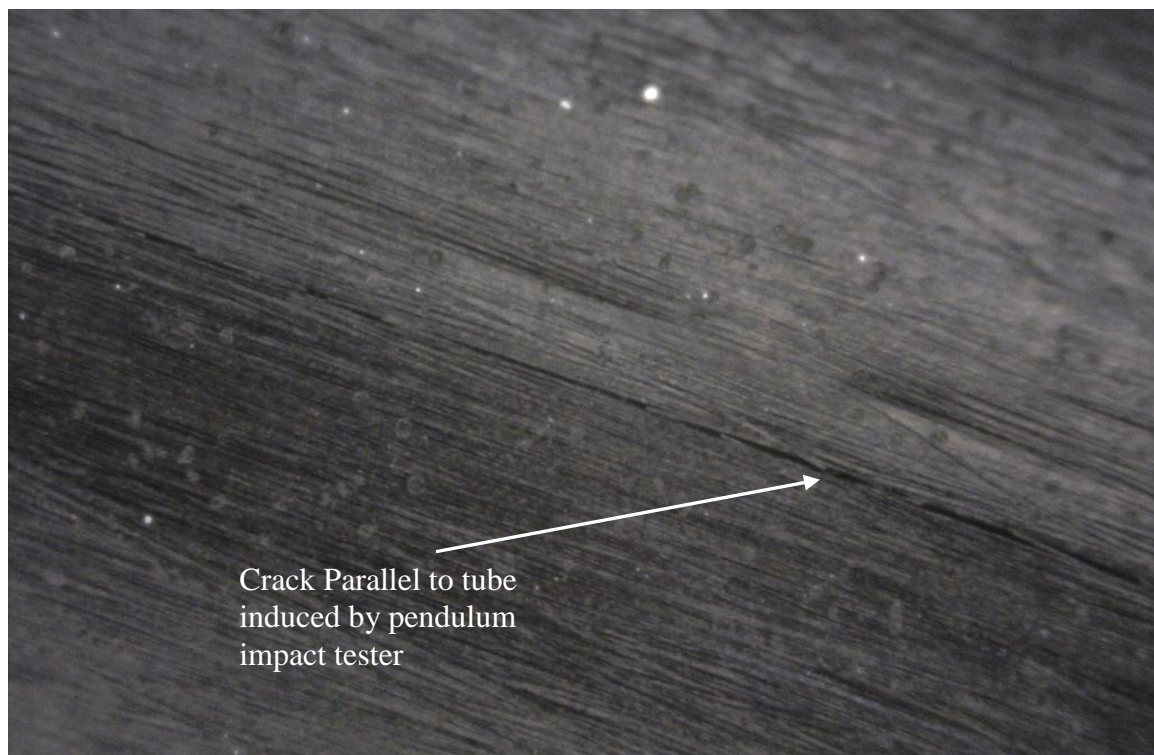


**Figure 17: Impact fixture**

Due to the scatter, it became necessary to accept an average reaction force which was within 10 N of the desired force.

### **Impact evaluation and fixture validation**

The data from these tests can be used for different purposes. Damage tolerance is an important area of concern which has been discussed at the beginning of this chapter. This test fixture combined with methods of examination can be used to evaluate damage tolerance. Upon close examination, it is possible to detect damage induced by the impact on the exterior layer. Unfortunately, the focus of this research is based upon damage which could be easily missed when using visual inspection methods, meaning it is not practical to evaluate the damage of interest through visual inspections. Figure 18 illustrates a portion of an A series tube which has been impacted. The area displayed





**Figure 18: A series tube with impact induced crack**

includes the side of the tube approximately 90 degrees from the actual site of impact and has been magnified for ease of viewing. From this magnified view it is possible to see cracks which run parallel with the tube axis between fibers.

Damage tolerance, however, focuses on the effects of pre-existing damage on the performance of the product. The selected method of evaluating damage tolerance is to perform fatigue tests on a combination of impacted and nonimpacted tubes. By comparing the resulting fatigue curves, trends can be found which correlate the degradation in fatigue life caused by the impact.

To illustrate the ability to evaluate damage tolerance using a combination of impacted and nonimpacted tubes, three fatigue curves were produced for five different tube designs. The first fatigue curve was produced using tubes which had not been impacted. The second curve used impacted tubes with a reaction force of approximately 315 N, and the third curve utilized tubes impacted with a reaction force of approximately 370 N. These reaction impact forces were predetermined based upon the characteristics of the tubes being used to validate the test fixture's effectiveness. Graphs were made which superimposed each of the three fatigue curves to illustrate the effect of the impact on the product life. The trends or effects of interest were noticed by the vertical shift in the corresponding fatigue curves.

Figure 19, Figure 20, and Figure 21 illustrate the damage tolerance of different tube designs with similar effective stiffness and average normal impact forces. Figures 21, 22, and 23 were produced using tubes of the same materials and design layup but

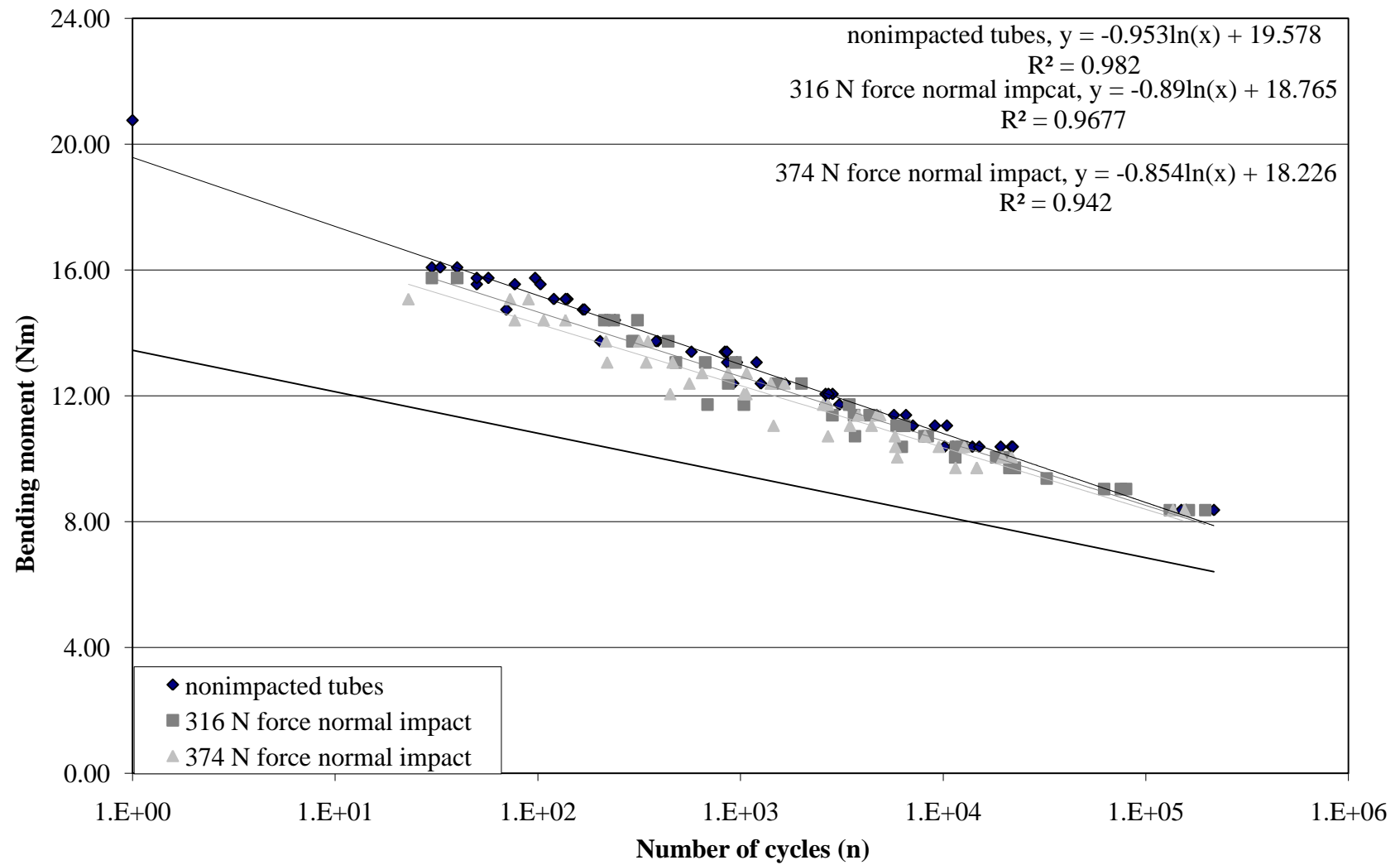
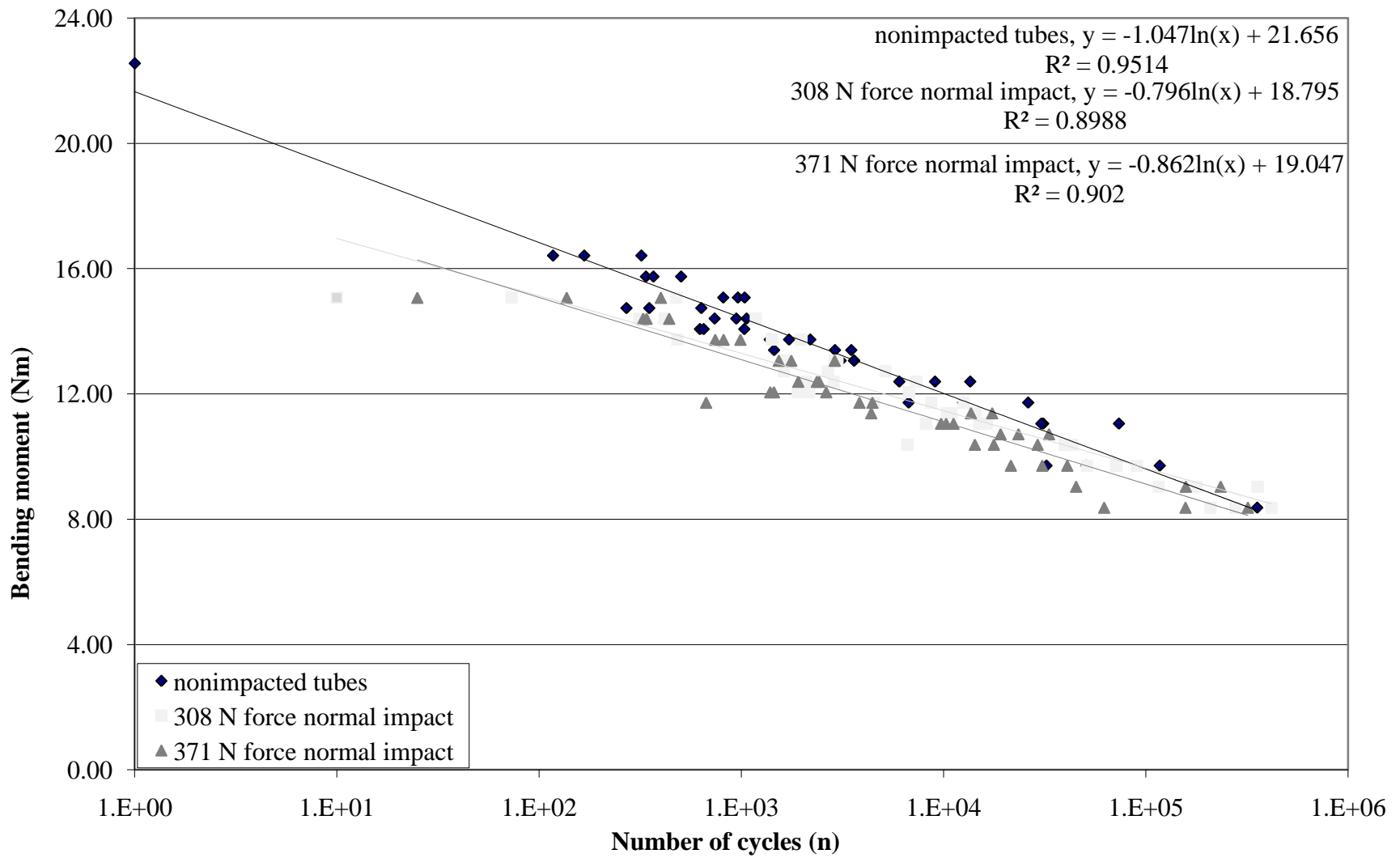


Figure 19: Fatigue curve series A,  $EI = 6.3 \text{ N m}^2$



**Figure 20: Fatigue curve series D,  $EI = 6.3 \text{ N m}^2$**

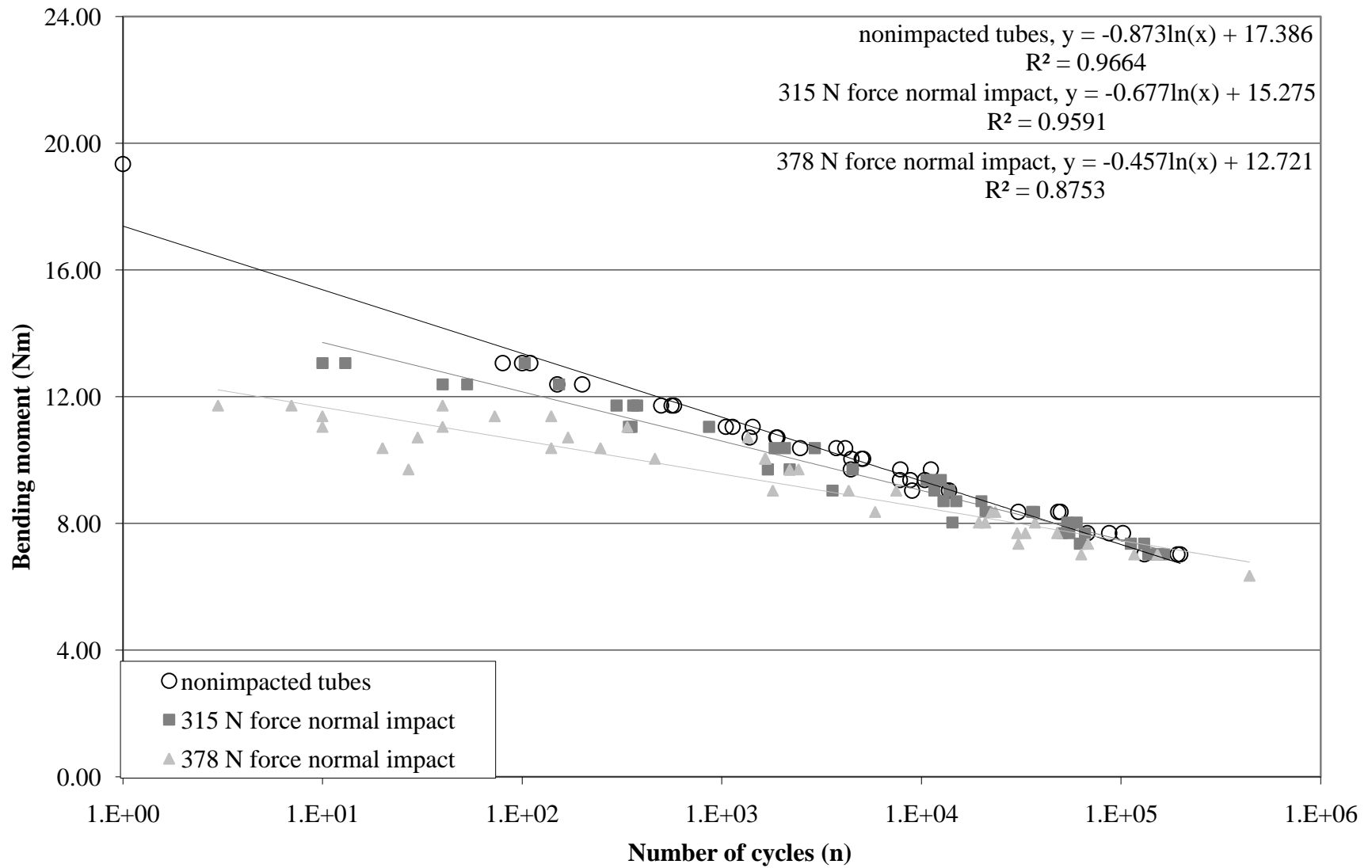


Figure 21: Fatigue curve series H,  $EI = 6.2 \text{ N m}^2$

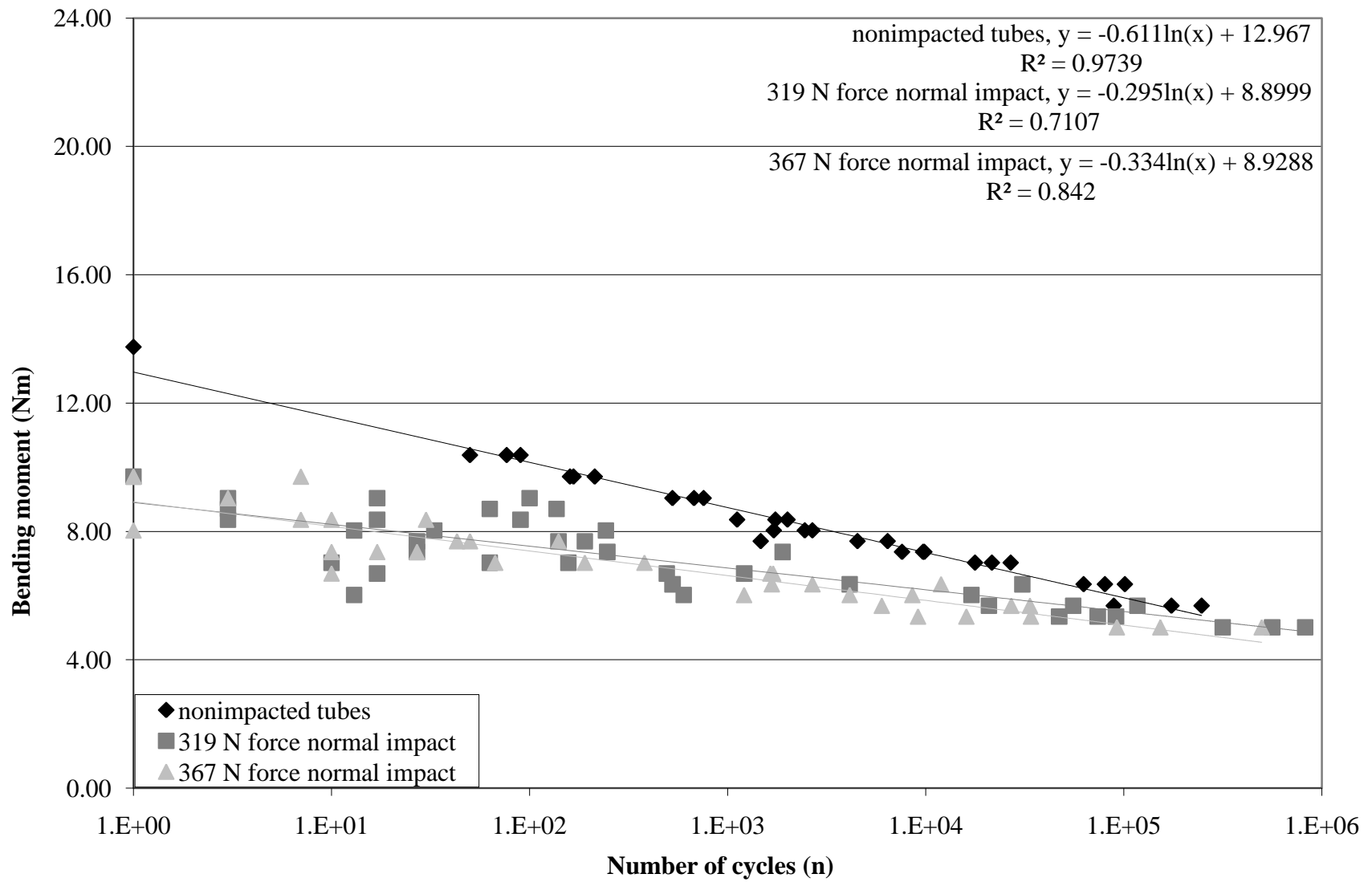


Figure 22: Fatigue curve series I,  $EI = 3.69 \text{ N m}^2$

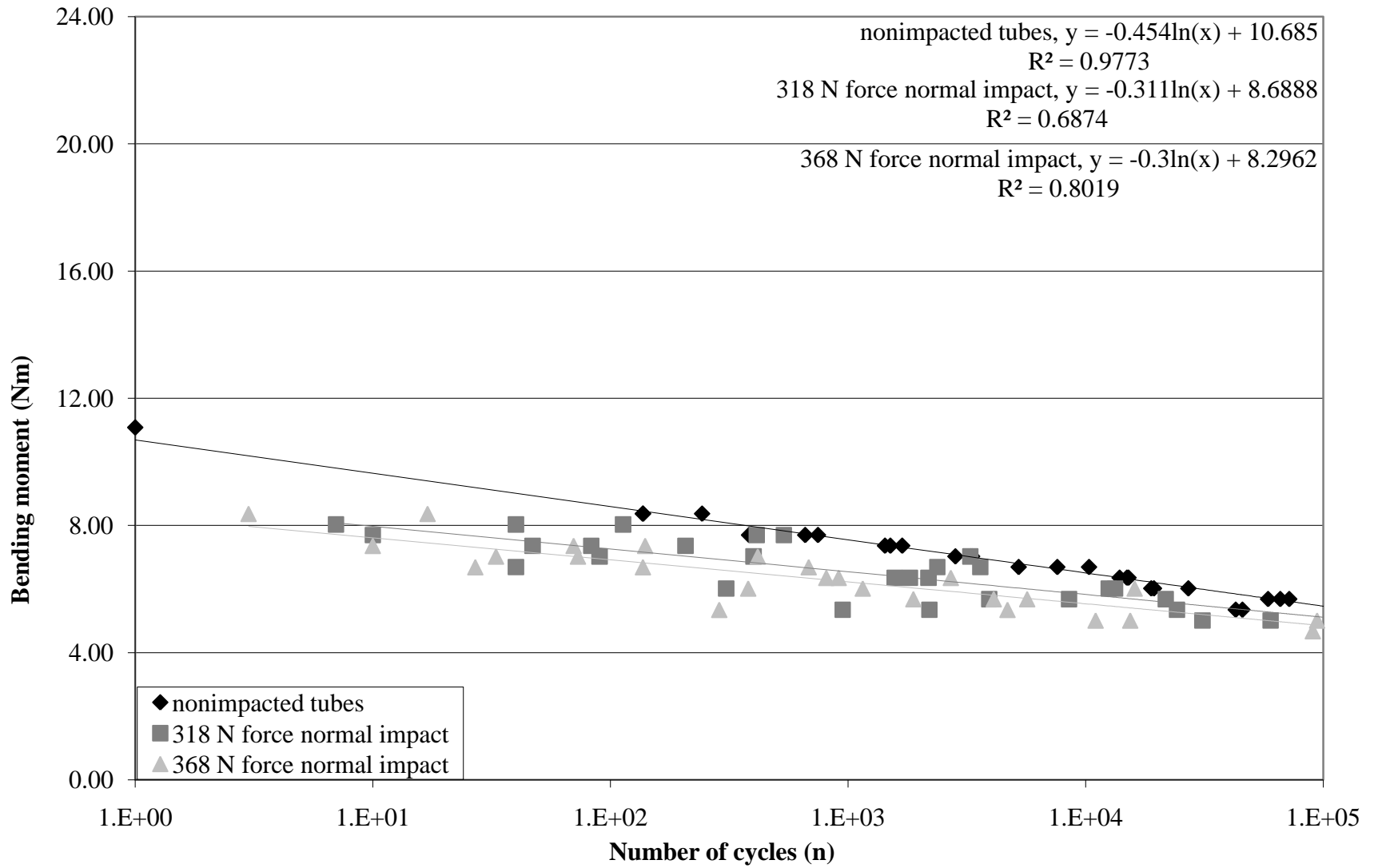


Figure 23: Fatigue curve series J,  $EI = 2.55 \text{ N m}^2$

have a different effective stiffness. They also used similar impact forces to generate the damage. These results were produced not only to see the how the effective stiffness affects the damage tolerance but also to illustrate the equipment's ability to test tubes of varying strength and stiffness. All impacts produced minimal visible damage, yet there were noticeable and varying effects on fatigue life. These tests suggest that the impact fixture is capable of providing the desired impact and corresponding product degradation necessary to evaluate tubes of different dimensions and effective stiffness.

## **CHAPTER 5**

### **CONCLUSION**

Testing methods developed in the previous three chapters are suitable methods for evaluating flexural properties of thin-walled composite tubes with a flexural stiffness between  $2.5 \text{ N-m}^2$  and  $9 \text{ N-m}^2$ . The four-point flexural test fixture is able to quickly evaluate the flexural strength of tubes with a flexural stiffness in the stated range. Following impacting, the fatigue fixture is capable of evaluating the product's damage tolerance. The impact fixture provides a method of producing damage induced by a glancing blow to the tube wall. By comparing reaction force to the tube deformation, it is possible to develop a better understanding of the tube's reaction to damage induced. The combination of these test methods provides a better understanding of product durability and design. Understanding of flexural strength, flexural fatigue performance, and damage tolerance under flexural fatigue loading provides a good reference or guideline to how durable tubes will be in actual use. Understanding these properties is necessary when deciding which tube designs are best suited for a given application.

Evaluating flexural properties of thin-walled composite tubes requires special attention to load application system for both quasi-static and fatigue testing. Special attention must be paid to the load applicator system because the tubes of interest have relatively thin walls. Rubber load applicators make it is possible to distribute necessary loads more uniformly without damaging the wall or deforming the tube into an oval



shape. The use of rubber load applicators makes it possible to transfer sufficient loads to induce failures from the applied bending moment without causing localized damage.

Incorporating the four-point flexural fixture provides the desired bending moment with two important aspects. The first is the need to minimize the applied loads while producing the maximum bending moment. The second is the presence of a region with a constant bending moment. These features made it possible to test critical sections of a tube such as those with induced damage. For this reason, the four-point flexural fixture with rubber load applicators is believed to be a suitable method of testing the flexural strength of thin-walled composite tubes.

Though there are many different methods of evaluating fatigue performance, the use of a rotating four-point fatigue fixture is believed to be well suited for this application. This test methodology was found to be a good match for the needs of this project which include the ability to test thin-walled composite tubes with a relatively low flexural stiffness and strength, the ability to test a specific region of a tube, and the ability to test all angle orientations around the tube axis. A method of testing all angle orientations around the tube axis is to rotate the tube during or between cycles of the test, which led to the concept of a rotating fatigue fixture. The other requirements are satisfied effectively by incorporating the four-point flexural test concept.

The concept of producing suitable forms of impact damage was a particularly difficult section of the project. Most impacters are intended to apply concentrated loads on a flat specimen. This approach applied to thin-walled tubes usually causes localized failures which are easily noticed and leave the tube completely unable to perform its intended task. Due to the fact that the tubes are inspected between cycles or operations, it

in unlikely that such tubes would be reused. However, short cracks which run parallel to the tube fiber on the outer layer are easy to miss during inspection. Thus, this type of damage was identified as the focus for this study of thin-walled composite tubes. The difficulty in studying this form of damage is that no available impacters are able to consistently produce such damage. A new impacter was needed which would produce this damage.

It is believed that a suitable method for introducing this damage is to focus on the deformation which can introduce such cracks. In this method, the tube is deformed into an oval shape gradually without applying large shear stresses at the edge of the impacting head. The methods which seemed most appropriate and produced damage similar to that induced in actual use was provided by an anvil on a pendulum arm. The arc of a pendulum gradually increased the deformation until it reached the desired maximum deformation and then gradually reduces the deformation. By using a pendulum which travels in an arc parallel to the tube axis, the impacter is capable of providing the desired tube deformation.

It is believed that combining these three test methods provides a better understanding of tube properties and durability. Evaluating the fatigue properties of tubes which have been impacted by the impact fixture makes it possible to see the amount of performance degradation caused by the impact. Superimposing fatigue curves created using impacted tubes on the original fatigue curve produced using nonimpacted tubes provides a clear view of the amount of tube life degradation and the rate of degradation for each tube design.

## APPENDIX

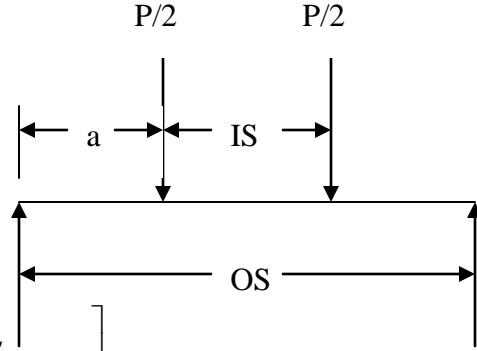
### Singularity Evaluation Method

$$\zeta = \frac{Pa}{EI} \left[ \frac{a^2}{6} + \frac{a * b}{4} + \frac{b^2}{16} \right]$$

$$a = \frac{OS - IS}{2}$$

$$b = IS$$

[1] Deflection equation of four-point bend from ASTM standards



$$\zeta = \frac{P}{EI} \left( \frac{OS - IS}{2} \right) \left[ \frac{\left( \frac{OS - IS}{2} \right)^2}{6} + \frac{OS - IS}{4} * IS + \frac{IS^2}{16} \right]$$

$$\zeta = \frac{P}{EI} \left( \frac{OS - IS}{2} \right) \left[ \frac{OS^2 - 2OS * IS + IS^2}{24} + \frac{(OS - IS)IS}{8} + \frac{IS^2}{16} \right]$$

$$\zeta = \frac{P}{EI} \left( \frac{OS - IS}{2} \right) \left[ \frac{OS^2}{24} - \frac{2OS * IS}{24} + \frac{IS^2}{24} + \frac{OS * IS}{8} - \frac{IS^2}{8} + \frac{IS^2}{16} \right]$$

$$\zeta = \frac{P}{EI} \left( \frac{OS - IS}{2} \right) \left[ \frac{OS^2}{24} - \frac{2OS * IS}{24} + \frac{IS^2}{24} + \frac{3OS * IS}{24} - \frac{2IS^2}{16} + \frac{IS^2}{16} \right]$$

$$\zeta = \frac{P}{EI} \left( \frac{OS - IS}{2} \right) \left[ \frac{OS^2}{24} + \frac{2IS^2}{48} + \frac{OS * IS}{24} - \frac{3IS^2}{48} \right]$$

$$\zeta = \frac{P}{EI} \left( \frac{OS - IS}{2} \right) \left[ \frac{OS^2}{24} + \frac{OS * IS}{24} - \frac{IS^2}{48} \right]$$

$$\zeta = \frac{P}{EI} \left[ \frac{OS^3}{48} + \frac{OS^2 * IS}{48} - \frac{OS * IS^2}{96} - \frac{OS^2 * IS}{48} - \frac{OS * IS^2}{48} + \frac{IS^3}{96} \right]$$

$$\zeta = \frac{P}{EI} \left[ \frac{OS^3}{48} - \frac{OS * IS^2}{96} - \frac{OS * IS^2}{48} + \frac{IS^3}{96} \right]$$

$$\zeta = \frac{P}{EI} \left[ \frac{OS^3}{48} - \frac{OS * IS^2}{96} - \frac{2OS * IS^2}{96} + \frac{IS^3}{96} \right]$$

$$\zeta = \frac{P}{EI} \left[ \frac{OS^3}{48} - \frac{3OS * IS^2}{96} + \frac{IS^3}{96} \right]$$

$$\zeta = \frac{P}{16EI} \left[ \frac{OS^3}{3} - \frac{OS * IS^2}{2} + \frac{IS^3}{6} \right]$$

Equation 2: Deflection equation [4]

## REFERENCES

- [1] ASTM D790, "Standard Test Methods for Flexural Properties of Unreinforced and Reinforced Plastics and Electrical Insulating Materials," American Society for Testing and Materials, West Conshohocken, PA, 2007
- [2] R. C. Hibbeler, *Mechanics of Materials*, fifth edition, Pearson Education, Inc. Upper Saddle River, New Jersey
- [3] <http://asm.matweb.com/search/SpecificMaterial.asp?bassnum=M434AE>